Ray and Wave Optics

Fill in the Blanks

Q.1. A light wave of frequency 5 × 10^{14} Hz enters a medium of refractive index 1.5. In the medium the velocity of the light wave is .... and its wavelength is .... (1983 - 2 Marks)

Ans. 2×10^8 m/s, 0.4×10^{-6} m

Solution.

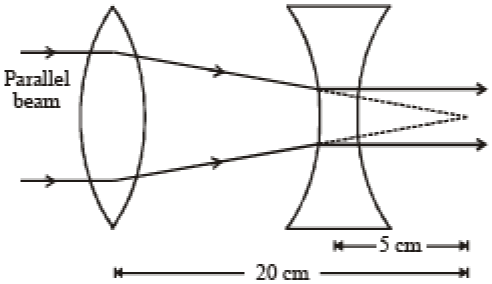
$$V_2 = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m/s};$$

$$\lambda_2 = \frac{V_2}{v} = \frac{2 \times 10^8}{5 \times 10^{14}} = 0.4 \times 10^{-6} m$$

Q.2. A convex lens A of focal length 20 cm and a concave lens B of focal length 5 cm are kept along the same axis with a distance d between them. If a parallel beam of light falling on A leaves B as a parallel beam, then d is equal to ...... cm. (1985 - 2 Marks)

Ans. d = +15 cm

Solution.



From the diagram it is clear that the focus of both the lenses should coincide as shown in the diagram.

Therefore d = 15 cm.

Q.3. A monoch romatic beam of light of wavelength 6000 Å in vacuum enters a medium of refractive index 1.5. In the medium its wavelength is ....., its frequency is ...... (1985 - 2 Marks)

Ans. 4000Å, 5×10^{14} Hz

Solution. KEY CONCEPT :

$$ {}_m^a\mu = \frac{\text{Speed of light in med 1}}{\text{Speed of light in med 2}} = \frac{\nu\lambda_a}{\nu\lambda_m} = \frac{\lambda_a}{\lambda_m} $$

[∴ v does not change with the medium]

$$ \therefore \quad \lambda_m = \frac{\lambda_a}{{}_m^a\mu} = \frac{6000}{1.5} = 4000\text{Å} $$

$$ \therefore \quad \nu_a = \frac{c_a}{\lambda_a} = \frac{3 \times 10^8}{6000 \times 10^{-10}} = 5 \times 10^{14}\,\text{Hz} $$

Q.4. In Young's double-slit experiment, the two slits act as coherent sources of equal amplitude 'A' and of wavelength 'λ'. In another experiment with the same set-up the two slits are sources of equal amplitude 'A' and wavelength 'λ', but are incoherent. The ratio of the intensity of light at the midpoint of the screen in the first case to that in the second case is .... (1986 - 2 Marks)

Ans. 2

Solution. For coherent sources, for constructive interference The amplitude at the mid point = A + A = 2A

$\Rightarrow I_1 \propto (2A)^2 \Rightarrow I_2 \propto 4\, I_0$ ... (i)

NOTE : For incoherent sources, the intensity add up normally (no interference).

Therefore, the total intensity I, = $2I_0$ ... (ii)

From (i) and (ii)

$$ \frac{I_1}{I_2} = \frac{4I_0}{2I_0} = 2 $$

Q.5. A thin lens of refractive index 1.5 has a focal length of 15 cm in air. When the lens is placed in a medium of refractive index 4/3, its focal length will become ....cm. (1987 - 2 Marks)

Ans. 60 cm

Solution.

$${}^{m}_{g}\mu = \frac{{}_{g}\mu}{{}_{m}\mu} = \frac{1.5}{4/3} = 1.125$$

$$\frac{1}{15} = (1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

[Lensmaker's formula]

$$\text{and } \frac{1}{f_2} = (1.125-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

On dividing we get

$$\frac{f_2}{15} = \frac{1.5-1}{1.125-1} = \frac{0.5}{0.125} = 4 \quad \therefore \quad f_2 = 60\text{ cm}$$

Q.6. A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at a distance of 9 meters and 25 meters respectively from the source. The ratio of amplitudes of the waves at P and Q is .... (1989 - 2 Marks)

Ans. 25/9

Solution.

$$\frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} \qquad \text{...(i)}$$

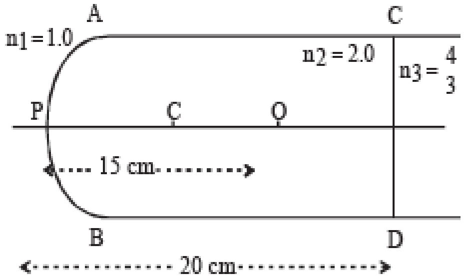
$$\text{But } I \propto \frac{1}{r^2} \Rightarrow \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \qquad \text{...(ii)}$$

From (i) and (ii),

$$\frac{A_1}{A_2} = \frac{r_2}{r_1} = \frac{25}{9}.$$

Q.7. A slab of a material of refractive index 2 shown in fig. has a curved surface APB of radius of curvature 10 cm and a plane surface CD. On the left of APB is

air and on the right of CD is water with refractive indices as given in the figure. An object O is placed at a distance of 15 cm from the pole P as shown. The distance of the final image of O from P, as viewed from the left is .... (1991 - 2 Marks)



Ans. 30 cm

Solution. For refraction at APB

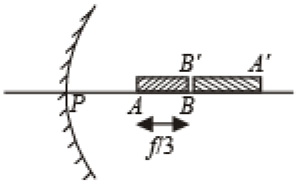
$$-\frac{\mu_2}{u}+\frac{\mu_1}{v}=\frac{\mu_1-\mu_2}{R}$$

⇒ Image of O will be formed at 30 cm to the right at P.

Q.8. A thin rod of length f/3 is placed along the optic axis of a concave mirror of focal length f such that its image which is real and elongated, just touches the rod. The magnification is .... (1991 - 1 Mark)

Ans. 1.5

Solution. Since the image formed is real and elongated, the situation is as shown in the figure. Since the image of B is formed at B' itself



∴ B is situated at the centre of curvature that is at a distance at 2f from the pole.

$$\therefore \quad PA = 2f-\frac{f}{3}=\frac{5f}{3}$$

Let us find the image of A. For point A, $u = -\frac{5f}{3}, v = ?$

$$\text{Applying, } \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{\frac{-5f}{3}} + \frac{1}{v} = \frac{1}{-f}$$

$$\Rightarrow \frac{1}{v} = -\frac{1}{f} + \frac{3}{5f} \Rightarrow v = -2.5f$$

Image length = 2.5 f – 2f = 0.5f

$$\therefore \text{ Magnification } = \frac{0.5f}{f/3} = 1.5$$

Q.9. A ray of light undergoes deviation of 30° when incident on an equilateral prism of refractive index √2. The angle made by the ray inside the prism with the base of the prism is .... (1992 - 1 Mark)

Ans. zero

Solution.

$$\mu = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin A/2}, \quad \sqrt{2} = \frac{\sin\left(\frac{60+\delta m}{2}\right)}{\sin 60/2}$$

$$\therefore \quad \frac{60+\delta m}{2} = 45° \Rightarrow \delta m = 30°$$

⇒ The condition is for minium deviation. In this case the ray inside the prism becomes parallel to base. Therefore the angle made by the ray inside the prism with the base of the prism is zero.

Q.10. The resolving power of electron microscope is higher that that of an optical microscope because the wavelength of electrons is .... than the wavelength of visible light. (1992 - 1 Mark)

Ans. smaller

Solution. KEY CONCEPT : The resolving power of a microscope device is inversely proportional to the wavelength used.

⇒ Ther esolving power of an electr on microscope is higher than that of an optical microscope because the wavelength of electrons is smaller than the wavelength of visible light.

Q.11. If ε_0 and μ_0 are, respectively, the electric permittivity and magnetic permeability of free space, ε and μ the corresponding quantities in a medium, the index of refraction of the medium in terms of the above parameters is .... (1992 - 1 Mark)

Ans. $\frac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0\varepsilon_0}}$

Solution. Velocity of light in vacuum $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ and the velocity of light in a medium $v = \frac{1}{\sqrt{\mu\varepsilon}}$

$$n = \frac{\text{Velocity light in vacuum}}{\text{Velocity light in medium}} = \frac{c}{v} = \frac{1/\sqrt{\mu_0\varepsilon_0}}{1/\sqrt{\mu\varepsilon}} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0\varepsilon_0}}$$

Q.12. A light of wavelength 6000Å in air, enters a medium with refractive index 1.5 Inside the medium its frequency is .... Hz and its wavelength is .... Å. (1997 – 2 Marks)

Ans. 5×10^{14} Hz, 4000Å

Solution. Frequency remains the same

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{6000 \times 10^{-10}} = 5 \times 10^{14}\,\text{Hz}$$

$$\text{and} \quad \lambda_2 = \frac{\lambda_1}{\mu} = \frac{6000\text{Å}}{1.5} = 4000\text{Å}$$

Q.13. Two thin lenses, when in contact, produce a combination of power +10 diopters. When they are 0.25 m apart, the power reduces to +6 diopters. The focal length of the lenses are .... m and ... m. (1997 - 2 Marks)

Ans. 0.125 m, 0.5 m

Solution. $P_1 + P_2 = 10\ m^{-1}$

$P_1 + P_2 – (0.25) P_1P_2 = 6m^{-1}$

From these two expressions, we get

$P_1 P_2 = 16m^{-2}$

$$P_1 - P_2 = \sqrt{(P_1 + P_2)^2 - 4P_1P_2}$$

$$= \sqrt{(10^{-1})^2 - 4(16^{-1})} = 6m^{-1}$$

$\therefore P_1 = 8m^{-1}$ and $P_2 = 2\ m^{-1}$, Hence

$$f_1 = \frac{1}{P_1} = \frac{1}{8}m = 0.125\,m \text{ and } f_2 = \frac{2}{P_2} = \frac{1}{2}m = 0.5m$$

Q.14. A ray of light is incident normally on one of the faces of a prism of apex angle 30° and refractive index √2 . The angle of deviation of the ray is... degrees.

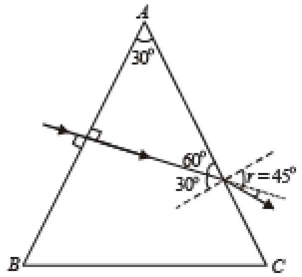
Ans. 15°

Solution. Using Snell's law for the refraction at AC, we get μ sin i = (1) sin r

√2 sin 30°= sin r ⇒ r = 45°

Angle of deviation at face AC

= 45° – 30° = 15°



True/False

Q.1. The setting sun appears higher in the sky than it really is. (1980)

Ans. T

Solution. This is due to atmospheric refraction. The light coming from sun bends towards the normal. Therefore, sun appears higher.

Q.2. The intensity of light at a distance 'r' from the axis of a long cylindrical source is inversely proportional to 'r'. (1981- 2 Marks)

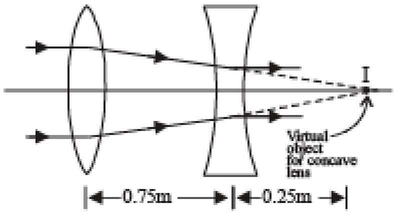
Ans. T

Solution. KEY CONCEPT : Formula for intensity of a line source of power (P) at a distance r from the source is $I = \frac{P}{2\pi r l}$

Q.3. A convex lens of focal length 1 meter and a concave lens of focal length 0.25 meter are kept 0.75 meter apart. A parallel beam of light first passes through the convex lens, then through the concave lens and comes to a focus 0.5 m away from the concave lens. (1983 - 2 Marks)

Ans. F

Solution. The image formed by the convex lens at the focus of the concave lens. Therefore I will act as a virtual object for concave lens and angle will be formed at infinity.



Q.4. A beam of white light passing through a hollow prism give no spectrum. (1983 - 2 Marks)

Ans. T

Solution. NOTE : For the light to split, the material through which the light passes should have refractive index greater than 1.

Since the prism is hollow, we get no spectrum. The thickness of glass slabs through which the prism is made can be neglected.

Q.5. The two slits in a Young's double slit experiment are illuminated by two different sodium lamps emitting light of the same wavelength. No interference pattern will be observed on the screen. (1984- 2 Marks)

Ans. T

Solution. When the two slits of Young's double slit experiment are illuminated by two different sodium lamps, then the sources are not coherent and hence sustained interference pattern will not be achieved. It will change so quickly that there will be general illumination and hence interference pattern will not be observed.

Q.6. In a Young's double slit experiment performed with a source of white light, only black and white fringes are observed. (1987 - 2 Marks)

Ans. F

Solution. In Young's double slit experiment if source is of white light than the central fringe is white with coloured fringes on either side.

Q.7. A parallel beam of white light fall on a combination of a concave and a convex lens, both of the same meterial. Their focal lengths are 15 cm and 30 cm respectively for the mean wavelength in white light. On the other side of the lens system, one sees coloured patterns with violet colour at the outer edge. (1988 - 2 Marks)

Ans. T

Solution.

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\Rightarrow \quad \frac{1}{F}=\frac{1}{-15}+\frac{1}{30}=\frac{-2+1}{30} \quad \Rightarrow \quad F=-30\text{ cm.}$$

This combination behaves as a concave lens of focal length 30 cm.

Since $F_v < F_r$.

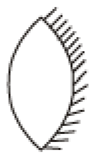
$\therefore$One sees coloured pattern with violet colour at the outer edge.

Subjective Questions Part -1

Q.1. A pin is placed 10 cm in front of a convex lens of focal length 20 cm, made a material of refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature are of 22 cm. Determine the position of the final image. Is the image real as virtual? (1978)

Ans. 11 cm, Real

Solution. The focal length of the equivalent mirror is



$$\frac{1}{F}=\frac{2}{f}+\frac{1}{f_m}$$

$$=\frac{2}{20}+\frac{2}{22}=\frac{1}{10}+\frac{1}{11}=\frac{21}{110}$$

$$\Rightarrow \quad F=\frac{110}{21}$$

NOTE : Since the focal length is positive it is a converging mirror

$$\text{Now, } \frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{-10}+\frac{1}{v}=\frac{1}{-110/21}$$

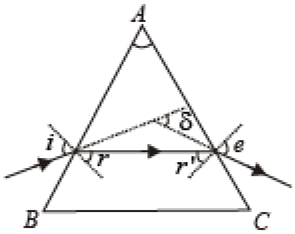
$$\Rightarrow \quad \frac{1}{v}=\frac{1}{10}-\frac{21}{110} \quad \Rightarrow \quad v=-11\text{cm}$$

NOTE : The negative sign indicates the image is real.

Q.2. A ray of light is incident at an angle of 60° on one face of prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of the prism. (1978)

Ans. 1.732

Solution. The situation can be shown as in the figure.



Here, i = 60°, A = 30°, d = 30°, e = ?

We know that, A + d = i + e ....(1)

Also, A = r + r' ....(2)

From (1), e = A + δ – i = 30° + 30° – 60° = 0

As the angle of emergence (e) is 0, hence the emergent ray is normal to the face from which it emerges.

When e = 0, r' = 0

∴ From (2), A = r = 30°.

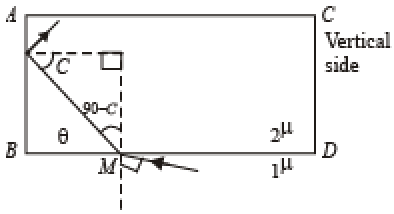
From Snell's law, refractive index of prism,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} = 1.732.$$

Q.3. A rectangular block of glass is placed on a printed page lying on a horizontal surface. Find the minimum value of the refractive index of glass for which the letters on the page are not visible from any of the vertical faces of the block. (1979)

Ans. 1.41

Solution.



For a grazing incident ray at BD for which i ≈ 90° the angle of refraction (90 – C) is maximum. For this C is least. Let C is greater than the critical angle.

Applying Snell's law at M

$$ {}_2^1\mu = \frac{\sin 90°}{\sin(90 - C)} \Rightarrow {}_2^1\mu = \frac{1}{\cos C} \qquad ...(i)$$

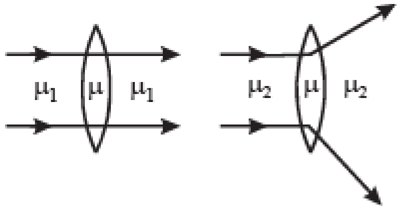
$$\text{Also } {}_2^1\mu = \frac{1}{\sin C} \qquad ...(ii)$$

When C is the critical angle.

$$\text{From (i) and (ii), } \frac{1}{\cos C} = \frac{1}{\sin C} \Rightarrow C = 45°$$

$$\therefore \quad {}_2^1\mu = \frac{1}{\sin 45°} = \sqrt{2} = 1.41$$

Q.4. What is the relation between the refractive indices μ_1 and μ_2, if the behaviour of light rays is as shown in the figure? (1979)



Ans. $\mu_1 < \mu_2$

Solution. For case (i), there is no refraction. Therefore $\mu_1 = \mu$

NOTE : Here the convex lens behaves as a diverging lens.

Therefore, $\mu < \mu_2$.

Q.5. An object is placed 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1.5 is then placed close to the mirror in the space between the object and the mirror.

Find the position of the final image formed. (You may take the distance of the near surface of the slab from the mirror to be 1 cm.

Ans. 7.67 cm

Solution. The rays or iginating from A (the point object) suffer refraction before striking the concave mirror.

For the mirror the rays are coming from A'

such that $AA' = \text{shift} = t\left(1 - \frac{1}{\mu}\right)$

Therefore the object distance

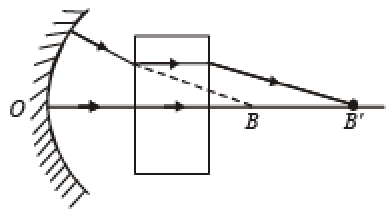
$$u = OA' = OA - AA' = 21 - t\left(1 - \frac{1}{\mu}\right)$$

$$= 21 - 3\left(1 - \frac{1}{1.5}\right) = 20\text{cm}$$

$$\therefore \quad v = \frac{uf}{u-f} = \frac{20 \times 5}{20-5} = \frac{20}{3}\text{cm} = 6.67\text{cm}$$

The reflected rays again pass through the glass slab. The image should have formed at B is the absence of glass slab.

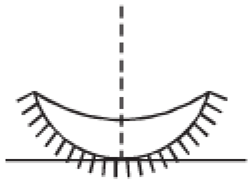
But. due to its presence the image is formed at B'.



Therefore image distance = OB + BB'

$$\frac{20}{3} + t\left(1 - \frac{1}{\mu}\right), \ \frac{20}{3} + 1 = \frac{23}{3} = 7.67\text{cm}$$

Q.6. The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface. (1981- 6 Marks)



(i) Where should a pin be placed on the optic axis such that its image is formed at the same place?

(ii) If the concave part is filled with water of refractive index 4/3, find the distance through which the pin should be moved so that the image of the pin again coincides with the pin.

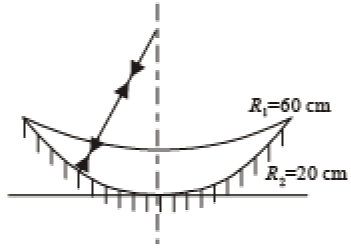
Ans. (i) 15 cm

(ii) 1.15 cm

Solution. KEY CONCEPT : The given silvered concavo-convex lens behaves like a mirror whose focal length can be calculated by the formula $\frac{1}{f} = \frac{2}{f_1} + \frac{1}{f_2}$

f_1 = focal length of concave surface.

f_2 = focal length of concave mirror



$$\therefore \quad \frac{1}{f} = \frac{2}{-30} + \frac{1}{-10} = -\frac{4}{30}$$

$$\therefore \quad f = -7.5 \text{ cm}$$

Using mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-7.5} = \frac{1}{-x} + \frac{1}{-x}$$

x = 15 cm

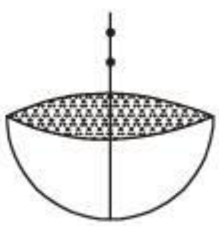
(ii) Let the object distance be u. When water is poured over the concave surface the apparent object distance will be v then

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

For flat surface R = ∞

$$\therefore \quad -\frac{\mu_1}{u} + \frac{\mu_2}{v} = 0$$

$$\Rightarrow \quad v = u\frac{\mu_2}{\mu_1} = u \times {}_{2}^{1}\mu = u \times \frac{4}{3}$$



Since the ray enters the lens from water into glass

$$\frac{-\mu_w}{u} + \frac{\mu_g}{v} = \frac{\mu_g - \mu_w}{R}$$

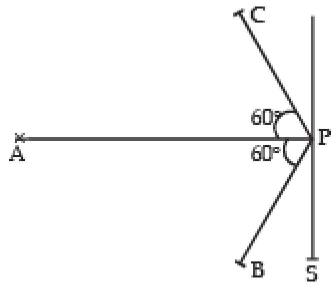
$$\Rightarrow \quad \frac{-4/3}{\frac{4}{3}u} + \frac{1.5}{-20} = \frac{1.5 - 4/3}{-60} \quad \Rightarrow \quad u = -13.85\,\text{cm}$$

∴ Downward shift = 15 – 13.85 = 1.15 cm.

Q.7. Screen S is illuminated by two point sources A and B. Another source C sends a parallel beam of light towards point P on the screen (see figure). Line AP is normal to the screen and the lines AP, BP and CP are in one plane. The distance AP, BP and CP are 3 m, 1.5 m and 1.5 m respectively. The radiant powers of sources A and B are 90 watts and 180 watts respectively. The beam

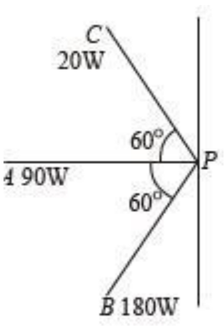
from C is of intensity 20 watts/m2.

Calculate the intensity at P on the screen. (1982 - 5 Marks)



Ans. 13.9

Solution. The total intensity at point P will be = $I_A + I_B + I_C$



$$I_A = \frac{(\text{Illumination power}) \times \cos\theta}{4\pi r^2}$$

$$= \frac{90 \times \cos 0}{4\pi \times 3^2}$$

$$= \frac{10}{4\pi} \text{watt/m}^2$$

$$I_B = \frac{180 \times \cos 60°}{4\pi \times (1.5)^2} = \frac{10}{\pi} \text{watt/m}^2$$

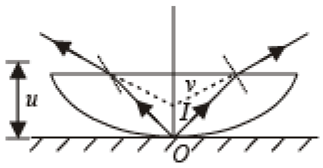
$$I_C = 20 \cos 60° = 10$$

$$\therefore \quad I_P = \frac{10}{4\pi} + \frac{10}{\pi} + 10 = 13.9 \text{W/m}^2$$

Q.8. A plano convex lens has a thickness of 4 cm . When placed on a horizontal table, with the curved surface in contact with it, the apparent depth of the bottom most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face is found to be 25/8 cm. Find the focal length of the lens. (1984- 6 Marks)

Ans. 75 cm

Solution. Here R = ∝ i.e., plane surface is the refracting surface



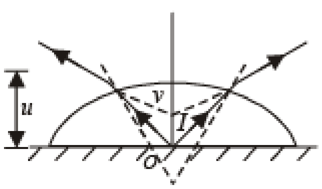
$$-\frac{\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R} \Rightarrow -\frac{\mu_1}{-4}+\frac{\mu_2}{-3}=0$$

$$\therefore \quad \frac{\mu_2}{\mu_1}=\frac{3}{4} \qquad \text{...(i)}$$

Again applying

$$-\frac{\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R} \Rightarrow -\frac{1}{u}+\frac{\mu_2/\mu_1}{v}=\frac{(\mu_2/\mu_1)-1}{R}$$

$$\Rightarrow \quad -\frac{1}{-4}+\frac{3/4}{-25/8}=\frac{3/4-1}{R}$$



On solving we get R = – 25cm.

Applying Len's maker formula,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{4}{3}-1\right)\left(\frac{1}{25}-\frac{1}{\infty}\right) \therefore f=75\text{cm}$$

Q.9. A beam of light consisting of two wavelengths, 6500Å and 5200Å, is used obtain interference fringes in a Young's double slit experiment : (1985 - 6 Marks)

(i) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 6500Å.

(ii) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

The distance between the slits is 2 mm and the distance between the plane of the slits and the screen is 120 cm.

Ans. (i) 1.17×10^{-3} m,

(ii) 1.56×10^{-3} m

Solution. (i) The distance of the nth bright fringe from the central maxima is given by the expression

$y_n = \frac{n\lambda D}{d}$, For 3rd bright fringe n = 3

$$\therefore \quad y = \frac{3 \times 6500 \times 10^{-10} \times 120 \times 10^{-2}}{2 \times 10^{-3}} = 1.17 \times 10^{-3} \text{m}$$

(ii) Let nth bright fringe of wavelength 6500 Å coincide with mth bright fringe of wavelength 5200Å. Their distance will be same from the central bright. Therefore,

$$\frac{n\lambda_1 D}{d} = \frac{m\lambda_2 D}{d} \qquad \therefore \quad \frac{n}{m} = \frac{5200}{6500} = \frac{4}{5}$$

i.e., at the least distance 4th bright fringe of 6500 Å will coincide with 5th bright fringe of 5200 Å. Its distance from the central maxima will be

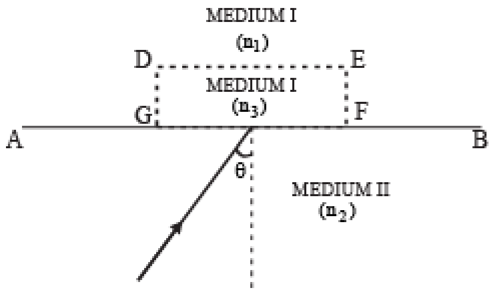
$$y_n = \frac{4 \times 6500 \times 10^{-10} \times 120 \times 10^{-2}}{2 \times 10^{3}} = 1.56 \times 10^{-3} \text{m}$$

Q.10. Monochromatic light is incident on a plane interface AB between two media of refractive indices n_1 and n_2 ($n_2 > n_1$) at an angle of incidence q as shown in fig. The angle θ is infinitesimally greater than the critical angle for the

two media so that total internal reflection takes place. Now if a transparent slab DEFG of uniform thickness and of refractive index n_3 is introduced on the interface (as shown in the figure), show that for any value of n_3 all light will ultimately be reflected back again into medium II. Consider separately the cases (1986 - 6 Marks)

(i) $n_3 < n_1$ and

(ii) $n_3 > n_1$.



Solution. KEY CONCEPT : For total internal reflection, the conditions are

The object should be in the denser medium.

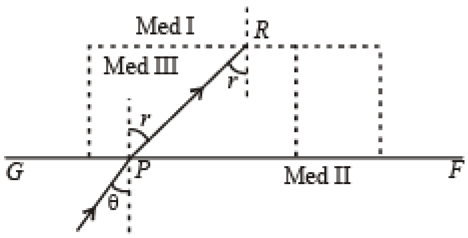
The angle of incidence should be greater than the critical angle

Case (i) : When $n_3 < n_1$

Obviously $n_3 < n_2$ and the angle θ is greater than the critical angle required for the ray passing from medium II to medium III. Therefore total internal reflection will also take place when a ray strikes with the same angle at the interface of medium II and medium III.

Case (ii) : $n_3 > n_1$ but $n_3 < n_2$

The ray will get refracted in medium III as the angle θ will now be less than the critical angle required for medium II and medium III pair.



$\therefore \quad \frac{\sin\theta}{\sin r} = \frac{n_3}{n_2}$ (Applying Snell's law at P)

$\therefore \quad \sin r = \frac{n_2}{n_3}\sin\theta$

As $n_2 > n_3$ So, $r > \theta$

When the refracted ray PR meets the boundary DE, it is travelling from a denser medium to a rarer medium. Therefore the ray will be totally internally reflected at DE if its angle of incidence r is more than the critical angle for med III and I.

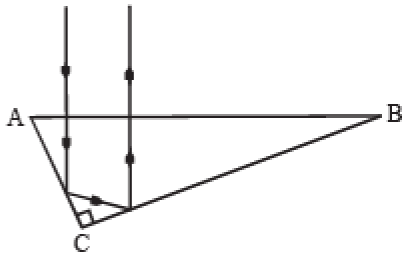
$\sin i'' = \frac{n_1}{n_3}$

Since, $\sin r > \frac{n_1}{n_3} \Rightarrow \sin r > \sin i'' \Rightarrow r > i''$

Therefore ray PR will be totally internal reflected along RQ.

On reaching Q, the ray will be refracted in med II. Thus, the ray will ultimately be reflected back in medium II.

Q.11. A right prism is to be made by selecting a proper material and the angles A and B (B $\leq$A) , as shown in Figure. It is desired that a ray of light incident on the face AB emerges parallel to the incident direction after two internal reflections.

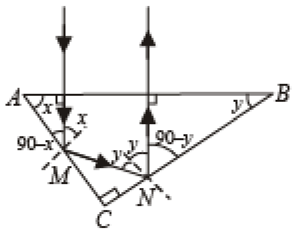


(i) What should be the minimum refractive index n for this to be possible ?

(ii) For n = 5/3 is it possible to achieve this with the angle B equal to 30 degrees ? (1987 - 7 Marks)

Ans. (i) $\sqrt{2}$ **(ii)** No

Solution. (i) Let x is the incident angle for reflection at AC. For total internal reflection x > i_C (critical angle)



Let y be the incident angle of the ray on face CB. For total internal reflection

internal reflection

$y > i_C$

$\therefore x + y > 2i_C$

But x = ∠A and y = ∠B (from geometry)

$\therefore x + y = 90°$

$\Rightarrow 90 > 2iC \Rightarrow i_C < 45°$

The refractive index of the medium for this to happen.

$$\mu = \frac{1}{\sin i_C} = \frac{1}{\sin 45°} = \sqrt{2}$$

(ii) $\mu = \frac{5}{3}$

$$\Rightarrow \sin i_C' = \frac{1}{\mu} = \frac{1}{5/3} = \frac{3}{5} \Rightarrow i_C' = 37°$$

$y = 30°$ (Given) $\therefore x = 60°$

$x > i_C'$ but $y < i_C'$

⇒ Total internal reflection will take place on face AC but not on CB.

Q.12. A parallel beam of light travelling in water (refractive index = 4/3) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial (1988 - 6 Marks)

(i) Find the position of the image due to refraction at the first surface and the position of the final image.

(ii) Draw a ray diagram showing the positions of both the images.

Ans. (i) –6 mm, –5 mm

Solution. (i) Initially the object is in denser medium and u = ∞ using the formula of refraction at a spherical surface for AB

$$-\frac{\mu_2}{u}+\frac{\mu_1}{v}=\frac{\mu_1-\mu_2}{R} \quad \Rightarrow \quad \frac{-4/3}{-\infty}+\frac{1}{v}=\frac{1-4/3}{2}$$

⇒ v = – 6 mm

NOTE : This is the position of the image due to refraction at the first surface. This image will behave as a virtual object for the refraction at the second surface.

u = – 6 – 4 = – 10 mm

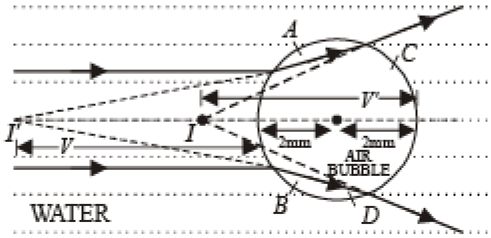
Again using the formula of refraction at a spherical surface for CD

$$-\frac{\mu_1}{u'}+\frac{\mu_2}{v'}=\frac{+\mu_2-\mu_1}{R}, \quad -\frac{1}{10}+\frac{4/3}{v'}=\frac{\frac{4}{3}-1}{-2}$$

⇒ v' = – 5 mm.

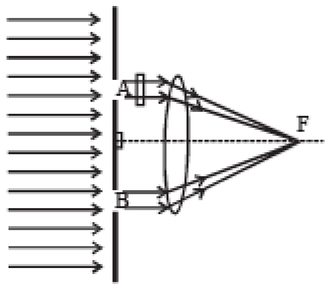
The is the position of final image.

(ii) Ray Diagram.



Q.13. In a modified Young's double slit experiment, a monochromatic uniform and parallel beam of light of wavelength 6000 Å and intensity (10/ π) W m^{-2} is incident normally on two circular apertures A and B of radii 0.001 m and 0.002 m respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength of 6000 Å is placed in front of aperture A, see fig. Calculate the power (in watts) received at the focal spot F of the lens.

The lens is symmetrically placed with respect to the apertures. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot. (1989 - 8 Mark)



Solution. The power transmitted through A

$$= \left[10\% \text{ of} \left(\frac{10}{\pi}\right)\right] \times \pi (0.001)^2 = 10^{-6}\,\text{W}$$

The power transmitted through B

$$= \left[10\% \text{ of} \left(\frac{10}{\pi}\right)\right] \times \pi \times (0.002)^2 = 4 \times 10^{-6}\,\text{W}$$

Let Δφ be the phase difference introduced by film

$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda}$ (path difference introduced by the film)

$$= \frac{2\pi}{\lambda} \times (\mu - 1)t = \frac{2\pi}{6000 \times 10^{-10}} [1.5 - 1] \times 2000 \times 10^{-10}$$

$$= \frac{\pi}{3} \text{ radian}$$

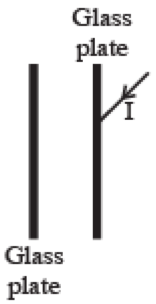
The power received at F

$$P = P_1 + P_2 + 2\sqrt{P_1 P_2} \cos \Delta\phi$$

$$= 10^{-6} + 4 \times 10^{-6} + 2\sqrt{10^{-6} \times 4 \times 10^{-6}} \cos \frac{\pi}{3}$$

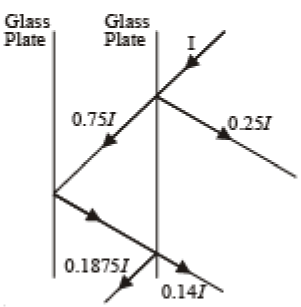
$$= 7 \times 10^{-6} \text{ W.}$$

Q.14. A narrow monochromatic beam of light of intensity I is incident on a glass plate as shown in figure. Another identical glass plate is kept close to the first one and parallel to it. Each glass plate reflects 25 per cent of the light incident on it and transmits the remaining. Find the ratio of the minimum and the maximum intensities in the interference pattern formed by the two beams obtained after one reflection at each plate. (1990 - 7 Mark)



Ans. 1/49

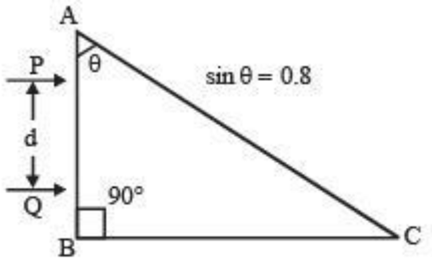
Solution. As shown in the figure, the interference will be between 0.25 I = I_1 and 0.14 I = I_2



$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}$$

$$= \frac{[\sqrt{0.25I}+\sqrt{0.14\,I}]^2}{[\sqrt{0.25I}-\sqrt{0.14\,I}]^2} = \frac{49}{1}.$$

Q.15. Two parallel beams of light P and Q (separation d) containing radiations of wavelengths 4000 Å and 5000 Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown in fig. The refractive index of the prism as a function of wavelength is given by the relation. $\mu(\lambda) = 1.20 + \frac{b}{\lambda^2}$ where λ **is in Å and b ispositive constant. The value of b is such that the condition for total reflection of the face AC is just satisfied for one wave length and is not satisfied for the other. (1991 - 2 + 2 + 4 Marks)**



(a) Find the value of b.

(b) Find the deviation of the beams transmitted through the face AC

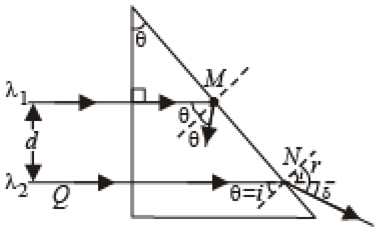
(c) A convergent lens is used to bring these transmitted beams into focus. If the intensities of transmission form the face AC, are 41 and I respectively, find the resultant intensity at the focus.

Ans. (a) 0.8×10^{-14} m^2

(b) 27.2°

(c) 9I

Solution. (a) λ_1 **= 4000 Å and** λ_2 **= 5000 Å**



For total internal reflection to take place, q should be greater than C. For smaller values of C, the values of m should be high or in other words the value of l should be small.

Therefore, total internal reflection will be given by

λ1 = 4000 Å

Here, sin θ = 0.8 (given) ⇒ θ = 53.1°

$$\therefore \quad \mu = \frac{1}{\sin\theta} = \frac{1}{0.8} = 1.25$$

$$\therefore \quad \mu = 1.2 + \frac{b}{(4000 \times 10^{-10})^2} = 1.25$$

$$\Rightarrow \quad b = 0.8 \times 10^{-14}\, m^2$$

(b) Applying Snell's law at N for wavelength λ_2

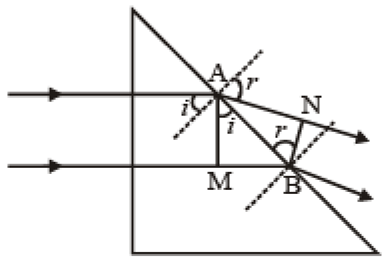
$$\mu = \frac{\sin r}{\sin i} \text{ where } \mu = 1.5 + \frac{0.8 \times 10^{-14}}{(5000 \times 10^{-10})^2} = 1.232$$

$$\Rightarrow \quad 1.232 = \frac{\sin r}{0.8} \Rightarrow r = 80.3°$$

From the figure it is clear that the deviation,

δ = r – i = 80.3° – 53.1° = 27.2°

(c) The intensities of transmitted beams are 4I and I respectively.



Path diff = $\mu(MB) - AN$

$$= \frac{\sin r}{\sin i}(AB\sin i) - AB\sin\theta$$

= 0

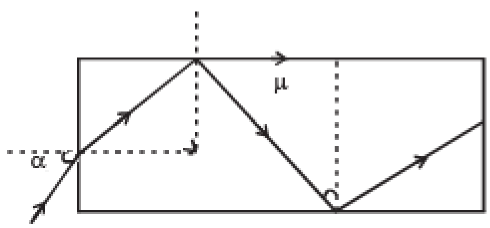
Since both the radiations are mutually coherent and while coming to focus these travel equal paths, therefore, these two beams will arrive in phase at focus.

∴ Resultant Intensity

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{4I} + \sqrt{I}\right)^2$$

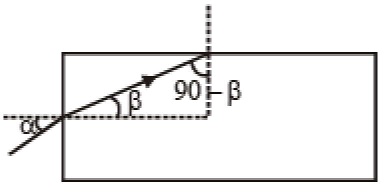
$$= (3\sqrt{I})^2 = 9I$$

Q.16. Light is incident at an angle α on one planar end of a transparent cylindrical rod of refractive index μ. Determine the least value of μ so that the light entering the rod does not emerge from the curved surface of rod irrespective of the value of μ (1992 - 8 Marks)



Ans. √2

Solution. The light entering the rod does not emerge from the curved surface of the rod when the angle (90° – β) is greater than the critical angle.



i.e., $\mu \le \frac{1}{\sin C}$ where C is the critical angle.

Here, C = 90 – β

$$\Rightarrow \mu \le \frac{1}{\sin(90° - \beta)} \Rightarrow \mu \le \frac{1}{\cos\beta}$$

As a limiting case, $\mu = \frac{1}{\cos\beta}$ ... (i)

Applying Snell's law at A

$$\mu = \frac{\sin\alpha}{\sin\beta} \Rightarrow \sin\beta = \frac{\sin\alpha}{\mu} \qquad ...(ii)$$

NOTE : The smallest angle of incidence on the curved surface is when α = π/2. This can be taken as a limiting case for angle of incidence on plane surface.

From (ii)

$$\sin\beta = \frac{\sin\pi/2}{\mu} \Rightarrow \mu = \frac{1}{\sin\beta} \qquad ...(iii)$$

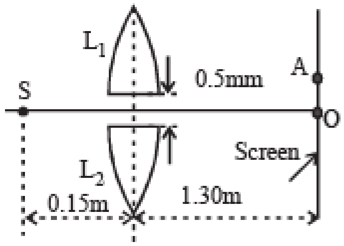
From (i) and (ii), sin β = cos β

⇒ β = 45°

$$\Rightarrow \mu = \frac{1}{\cos 45°} = \frac{1}{1/\sqrt{2}} \Rightarrow \mu = \sqrt{2}$$

This is the least value of the refractive index of rod for light entering the rod and not leaving it from the curved surface.

Q.17. In Fig., S is a monochromatic point source emitting light of wavelength λ = 500nm. A thin lens of circular shape and focal length 0.10 m is cut into two identical halves L_1 and $L_2$by a plane passing through a diameter. The two halves are placed symmetrically about the central axis SO with a gap of 0.5 mm. The distance along the axis from S to L_1 and L_2 is 0.15 m while that from L_1 and L_2 to O is 1.30 m. The screen at O is normal to SO. (1993 – 5+1 Marks)



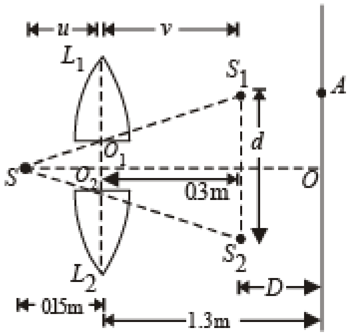
(i) If the third intensity maximum occurs at the point A on the screen, find the distance OA.

(ii) If the gap between L_1 and L_2 is reduced from its original value of 0.5mm, will the distance OA increase, decrease, or remain the same?

Ans. (i) 10^{-3} m

(ii) increase

Solution. (i) In this case, the two identical halves of convex lens will create two seperate images S_1 and S_2 of the source S. These Images (S_1 and S_2) will behave as two coherent sources and the further dealing will be in accordance to Young's double slit experiment.



For lens L_1

The object is S

u = – 0.15 m, v = ?, f = + 0.1 m

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{0.1}+\frac{1}{-0.15}$$

$\Rightarrow$ v = 0.3 m

$\Delta\ SO_1O_2$ and $\Delta\ SS_1S_2$ are similar. Also the placement of O_1 and O_2 are symmetrical to S

$$\therefore \quad \frac{S_1S_2}{O_1O_2}=\frac{u+v}{u}$$

$$\Rightarrow \quad S_1S_2=\frac{(u+v)(O_1O_2)}{u}=\frac{(0.15+0.3)}{(0.15)}\times 0.5\times 10^{-3}$$

$\Rightarrow S_1S_2 = d = 1.5 \times 10^{-3}$ m $\therefore$ D = 1.3 – 0.3 = 1m

The fringe width

$$\beta=\frac{\lambda D}{d}=\frac{500\times 10^{-9}\times 1}{1.5\times 10^{-3}}=\frac{1}{3}\times 10^{-3}\text{m}$$

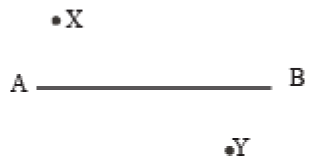
$\therefore$ Therefore,

$$OA=3\beta=3\times\frac{1}{3}\times 10^{-3}\text{m}=10^{-3}$$

(ii) If the gap between L_1 and L_2 i.e., O_1O_2 is reduced.

Then d will be reduced. Then the fringe width will increase and hence OA will increase.

Q.18. An image Y is formed of point object X by a lens whose optic axis is AB as shown in figure. Draw a ray diagram to locate the lens and its focus. If the image Y of the object X is formed by a concave mirror (Having the same axis as AB) instead of lens, draw another ray diagram to locate the mirror and its focus. Write down the steps of construction of the ray diagrams. (1994 - 6 Marks)



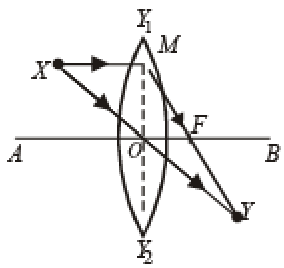
Solution. (i) Since Y is below of optic axis, therefore the image is real and inverted.

(i) STEPS OF CONSTRUCTION OF DIAGRAM.

For convex lens

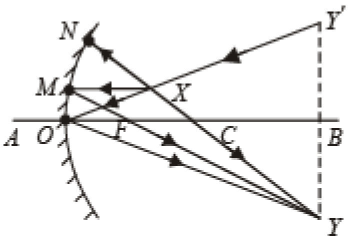
(1) Join XY. This represents the ray originating from the source and meeting the image Y. Since the ray is undeviated after passing through the lens, therefore O is the optical centre of the lens. Draw Y_1OY_2perpendicular to AB.

(2) Draw a ray from X, parallel to AB. It strikes Y_1OY_2 at M. Join MY. It cuts AB at F. This is the focus of the convex lens.



(ii) For concave mirror

As the image is real and inverted, the concave mirror has to be placed towards the left of X. **To find the exact position of the concave mirror**, we draw a line YY' perpendicular to AB such that BY = BY'



Join Y'X and extend the line to meet AB at O. If the concave mirror is placed at O then after reflection at O, this line will meet Y.

To find the radius of curvature of the mirror

Join X and Y. Let it cut AB at C. This C should be the centre of curvature of the concave mirror. With OC as radius, draw a part of sphere. This is the concave mirror.

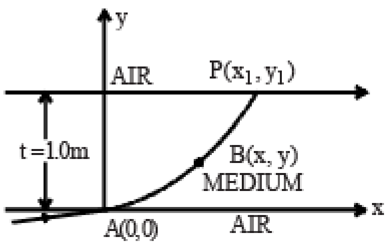
To find the focus of the concave mirror

Draw XM parallel to the principal axis. Join M to Y. Let it cut AD at F. Therefore, F is the focus of concave mirror.

Q.19. A ray of light travelling in air is incident at grazing angle (incident angle ≅ 90°) on a long rectangular slab of a transparent medium of thickness t = 1.0 m (see figure below).

The point of incidence is the origin A(0, 0). The medium has a variable index of refraction n(y) given by

$n(y) = [ky^{3/2} + 1]^{1/2}$, where k = 1.0 $(\text{metre})^{-3/2}$



The refractive index of air is 1.0. (1995 - 10 Marks)

(a) Obtain a relation between the slope of the trajectory of the ray at a pint B(x, y) in the medium and the incident angle at that point.

(b) Obtain an equation for the trajectory y(x) of the ray in the medium.

(c) Determine the coordinates (x_1 ,y_1) of the point P, where the ray intersects the upper surface of the slabair boundary.

(d) Indicate the path of the ray subsequently.

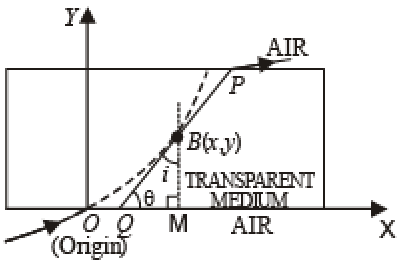
Ans. (a) $\frac{dy}{dx} = \cot i$

(b) $y = k^2\left(\frac{x}{4}\right)^4$

(c) (4m, 1m)

Solution. (a) SLOPE AT P

To find the slope at B, we draw a tangent to the trajectory at B. The trajectory is such that as the ray passes through the rectangular transparent medium, the ray continuously deviates towards the normal. The tangent at B makes an angle θ with the x-axis. Therefore, the slope at point B is tan $= \frac{dy}{dx}$ ... (i)



i is the angle of incidence at B then according to ΔBQM

$$i + \theta + \frac{\pi}{2} = \pi \quad \text{... (ii)}$$

Substituting the value of q from (ii) in (i)

$$\tan\left(\frac{\pi}{2} - i\right) = \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \cot i \quad \text{... (iii)}$$

(b) EQUATION OF TRAJECTORY

According to Snell's law, when light propagates through a series of parallel layers of different media, then n sin i = constant

Let us consider the rectangular state to be made up of parallel layers such that as we move in the + Y direction, the refractive index increases as given by the relationship

$n(y) = [ky^{3/2} + 1]^{1/2}$ ...(iv)

Applying Snell's law at O, we get 1 × sin 90° = constant = 1.

Again applying Snell's law at B, we get

n sin i = const. = 1 (from above equation)

$$\therefore \quad n = \frac{1}{\sin i} = \text{cosec } i = \sqrt{1+\cot^2 i} = \sqrt{1+\left(\frac{dy}{dx}\right)^2},$$

$$\sqrt{ky^{3/2}+1} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \quad \text{from (iv)}$$

$$\Rightarrow \quad \frac{dy}{dx} = [ky^{3/2}]^{1/2} \Rightarrow \frac{dy}{y^{3/4}} = k^{1/2}dx = dx \quad (\because k = 1)$$

$$\Rightarrow \quad \int \frac{dy}{y^{3/4}} = \int dx$$

$\Rightarrow 4_y^{1/4} = x + C$ where C is an integration constant.

But at x = 0, y = 0

$$\therefore \quad C = 0 \quad \therefore 4y^{1/4} = x \Rightarrow y = \left(\frac{x}{4}\right)^4$$

(c) CO-ORDINATES (x_1, y_1) OF THE POINT P

At P, y = 1m $\quad \therefore x = 4\, y^{1/4} = 4$

The coordinates of P are (4m, 1m)

(d) The refractive index at P

$n_p = [ky^{3/2} + 1]^{1/2} = [1\,(1)^{3/2} + 1]^{1/2} = \sqrt{2}$

If i_p is angle of incidence at P then according to Snell's law,

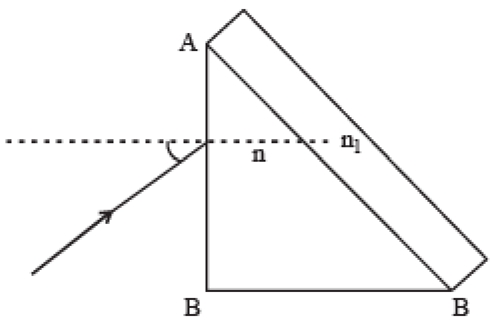
$$n_p \sin i_p = 1 \Rightarrow \sin i_p = \frac{1}{\sqrt{2}}$$

Also by Snell's law, $n_{air} \sin r_p = n_p \sin i_p$

$$1 \sin r_p = \sqrt{2} \times \frac{1}{\sqrt{2}} \Rightarrow \sin r_p = 1 \Rightarrow r_p = \frac{\pi}{2}$$

⇒ After emerging from the rectangular glass slab, the light ray becomes parallel to slab length.

Q.20. A right angled prism (45° –90°–45°) of refractive index n has a plate of refractive index n_1(n_1 < n) cemented to its diagonal face. The assembly is in air. A ray is incident on AB.



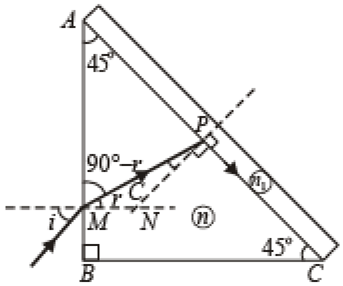
(i) Calculate the angle of incidence at AB for which the ray strikes the diagonal face at the critical angle.

(ii) Assuming n = 1.352 calculate the angle of incidence at AB for which the refracted ray passes through the diagonal face undeviated. (1996 - 3 Marks)

Ans. (i) $\sin^{-1}\left[\frac{1}{\sqrt{2}}\left\{\sqrt{n^2 - n_1^2} - n_1\right\}\right]$ (ii) 72.94°

Solution. (i) The ray incident on AB at M makes an angle of incidence i. It gets refracted at M. The angle of refraction is r. Applying Snell's law at M

$$n = \frac{\sin i}{\sin r} \qquad \text{...(i)}$$



From fig

∠APM = 180° – (45° + 90° – r) = 45° + r and C = 90° – (45° + r) = 45° – r

The ray after refraction at M enter the prism and strikes its diagonal face AC making an angle C with the normal at P.

Here C is the critical angle, therefore, the ray after refraction at P makes angle of refraction 90° Applying Snell's law at P

$$\frac{n}{n_1} = \frac{\sin 90°}{\sin C} \Rightarrow \sin C = \frac{n_1}{n} \qquad \text{... (ii)}$$

From (i), sin i = n sin r   = n sin (45° – C)

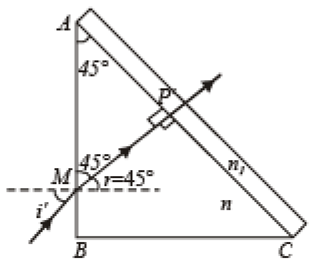
= n [sin 45° cos C – cos 45° sin C]

$$= \frac{n}{\sqrt{2}}[\sqrt{1-\sin^2 C} - \sin C]$$

$$\sin i = \frac{n}{\sqrt{2}}\left[\sqrt{1-\frac{n_1^2}{n^2}} - \frac{n_1}{n_2}\right] \qquad \text{[From (ii)]}$$

$$\Rightarrow \quad i = \sin^{-1}\left[\frac{1}{\sqrt{2}}\{\sqrt{n^2 - n_1^2} - n_1\}\right]$$

(ii) Angle of incidence at AB for which the refracted ray passes through the diagonal face undeviated. For this to happen, the angle of incidence of ray MP on diagonal face should be zero. It means that the ray should strike normal to AC.



Applying Snell's law at M, we get $n = \frac{\sin i'}{\sin r}$

Since ∠AP'M = 90° ∠ AMP = 45 ⇒ r = 45º

$\therefore \quad \sin i' = n \sin r = n \sin 45° = \frac{1.352}{\sqrt{2}} = 0.956$

⇒ i ' = 72.94°

Subjective Questions Part -2

Q.21. A double-slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1mm, and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å. (1996 - 3 Marks)

(i) Calculate the fringe-width.

(ii) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minimum on the axis.

Ans. (i) 6.3×10^{-4} m **(ii)** 1.575×10^{-6} m

Solution. (i) $_{m}^{a}\mu = \frac{\lambda_a}{\lambda_m} \Rightarrow \lambda_m = \frac{\lambda_a}{_{m}^{a}\mu}$

$\therefore$ Fringe width $= \frac{\lambda_a D}{_{m}^{a}\mu d} = \frac{6300 \times 10^{-10} \times 1.33}{1.33 \times 10^{-3}}$

$= 6.3 \times 10^{-4}$m

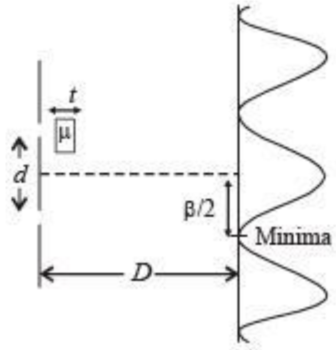
(ii) KEY CONCEPT : The shift of fringes when one slit is covered with thin glass sheet is

$$= \frac{Dt}{d}\left[\frac{_{g}\mu}{_{m}\mu} - 1\right]$$

where, t = thickness of glass sheet.

The shift has to be such that the minima shifts to the axis.

For this the shifting of the fringes should be β/2 where b is fringe width



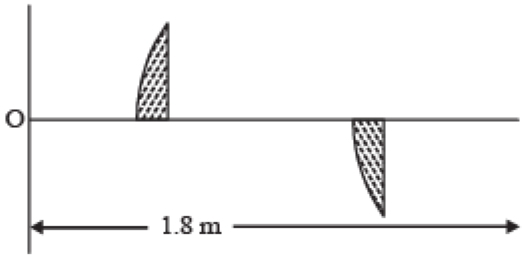
$$\therefore \quad \frac{Dt}{d}\left[\frac{{}_g\mu - {}_m\mu}{{}_m\mu}\right] = \frac{\beta}{2}$$

$$\Rightarrow \quad t = \frac{\beta d\, {}_m\mu}{2({}_g\mu - {}_m\mu) \times D} = \frac{6.3 \times 10^{-4} \times 10^{-3} \times 1.33}{2(1.53 - 1.33) \times 1.33}$$

$$= 15.75 \times 10^{-7}\,\text{m} = 1.575 \times 10^{-6}\,\text{m}$$

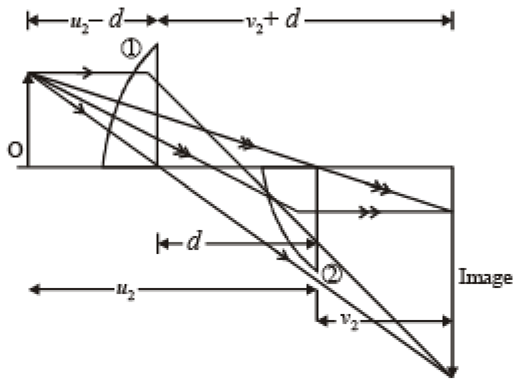
Q.22. A thin plano-convex lens of focal length f is split into two halves: one of the halves is shifted along the optical axis . The separation between object and image planes is 1.8 m.

The magnification of the image formed by one of the halflenses is 2. Find the focal-length of the lens and separation between the two halves. Draw the ray diagram for image formation. (1996 - 5 Marks)



Ans. 0.4m, 0.6m

Solution.



Given $u_2 + v_2 = 1.8$ m ... (i)

The magnification of lens (1) is 2

$$\therefore \quad 2 = \frac{v_2 + d}{u_2 - d} \qquad \text{...(ii)}$$

From (i) and (ii)

$u_2 = 0.6 + d$, $v_2 = 1.2 - d$

Applying lens formula

$$\frac{1}{v_2 + d} + \frac{1}{u_2 - d} = \frac{1}{f} \qquad \text{.....(iii)} \quad \text{for lens (1)}$$

$$\frac{1}{v_2} + \frac{1}{u_2} = \frac{1}{f} \qquad \text{... (iv)} \quad \text{for lens (2)}$$

From (iii) and (iv)

$$\frac{1}{v_2 + d} + \frac{1}{u_2 - d} = \frac{1}{v_2} + \frac{1}{u_2}$$

$$\Rightarrow \quad \frac{1}{1.2 - d + d} + \frac{1}{0.6 + d - d} = \frac{1}{1.2 - d} + \frac{1}{0.6 + d}$$

On solving, we get

$\Rightarrow d = 0.6$ m

Substituting this value in (iv)

$$\frac{1}{1.2-0.6}+\frac{1}{0.6+0.6}=\frac{1}{f}$$

$\therefore$ f = 0.4 m

Q.23. In Young's experiment, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate, having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å. It is found that the point P on the screen where the central maximum (n = 0) fells before the glass plates were inserted now has 3/4 the original intensity. It is further observed that what used to be the fifth maximum earlier, lies below the point P while the sixth minimum lies above P. Calculate the thickness of the glass plate. (Absorption of light by glass plate may be neglected.) (1997 - 5 Marks)

Ans. 9.3×10^{-6}m

Solution.

The phase difference $\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}(5\lambda + \Delta)$

We know that $I(\phi) = I_{max}\cos^2\left(\frac{\phi}{2}\right)$

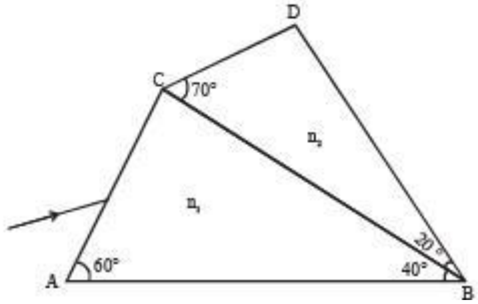
$$\Rightarrow \frac{3}{4}I_{max} = I_{max}\cos^2\frac{\phi}{2} \Rightarrow \frac{\phi}{2} = 30° = \frac{\pi}{6}$$

$$\Rightarrow \frac{2\pi}{6} = \frac{2\pi}{\lambda}(5\lambda + \Delta) \quad \Rightarrow \Delta x = \frac{\lambda}{6} = 0.3\,t$$

$$\Rightarrow t = 9.3 \times 10^{-6}\,\text{m}$$

Q.24. A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together without a gap as shown in Figure.

The angles of the prisms are as shown. n_1 and n_2 depend on λ, the wavelength of light, according to $n_1 = 1.20 + \frac{10.8 \times 10^4}{\lambda^2}$ **and** $n_2 = 1.45 + \frac{1.80 \times 10^4}{\lambda^2}$ **where λ is in nm. (1998 - 8 Marks)**



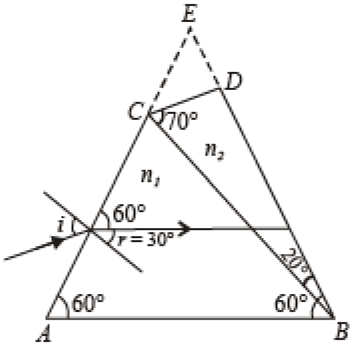
(a) Calculate the wavelength λ_0 for which rays incident at any angle on the interface BC pass through without bending at that interface.

(b) For light of wavelength λ_0, find the angle of incidence i on the face AC such that the deviation produced by the combination of prisms is minimum.

Ans. **(a)** 600 nm

(b) $\sin^{-1}\left(\frac{3}{4}\right)$

Solution.



For Minimum Deviation

(a) The rays of wavelength λ_0 incident at any angle on the interface BC will pass through without bending, provided the refractive indices n_1 and n_2 have the same value for the wavelength λ_0. Equating the expressions of n_1 and n_2, we get

$$1.20+\frac{10.8\times10^{-4}}{\lambda_0^2}=1.45+\frac{1.80\times10^{-4}}{\lambda_0^2}$$

(where λ_0 is in nm)

or $\lambda_0 = \left(\frac{9.0 \times 10^4}{0.25}\right)^{1/2} = 600\,\text{nm}$

(b) For the wavelength 600 nm, the combination of prism acts as a single prism shaped like an isosceles triangle (ABE). At the minimum deviation, the ray inside the prism will be parallel to the base. Hence, the angle of refraction on the face AC will be r = 30°.

Now sin i = n sin r = n sin 30° = n/2 ... (1)

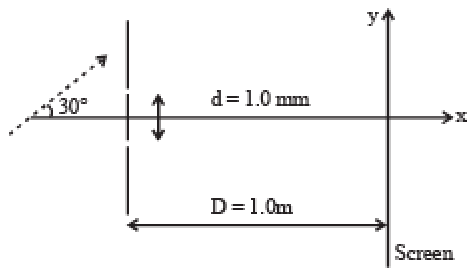
The value of n at 600 nm is

$$n = 1.20 + \frac{10.8 \times 10^4}{(600)^2} = 1.50 \qquad \text{... (2)}$$

From (1) and (2),

the angle of incidence is $i = \sin^{-1}\left(\frac{3}{4}\right)$

Q.25. A coherent parallel beam of microwaves of wavelength λ = 0.5 mm falls on a Young's double slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 m from it as shown in Fig.



(a) If the incident beam falls normally on the double slit apparatus, find the y-coordinates of all the interference minima on the screen.

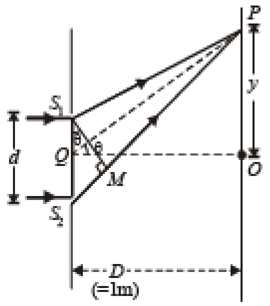
(b) If the incident beam makes an angle of 30° with the x axis (as in the dotted arrow shown in Figure), find the y-coordinate of the first minima on either side of the central maximum. (1998 - 8 Marks)

Ans. (a) $\pm$ 0.26 m, $\pm$1.13 m

(b) 1.13 m, 0.26 m

Solution. (a) The path difference (Δx) from the ray starting from S_1 and S_2 and reaching a point P will be

$\Delta x = d \sin\theta$



We know that the path difference for minimum intensity is

$(2m-1)\dfrac{\lambda}{2}$ where $m = 1, 2, 3...$

$\therefore \quad d\sin\theta = (2m-1)\dfrac{\lambda}{2}$

$\Rightarrow \quad \sin\theta = \dfrac{(2m-1)\lambda}{2d} = \dfrac{(2m-1)0.5}{2\times 1.0} = \dfrac{2m-1}{4}$

Also $-1 \le \sin\theta \le 1$. Therefore, possible values of m are $\pm$ 1, $\pm$ 2, 0

From Δ POQ

$$y = D\tan\theta = \frac{D\sin\theta}{\sqrt{1-\sin^2\theta}} \qquad \text{... (i)}$$

Positions of minima

For $m = +1$, $\sin\theta = \dfrac{1}{4}$ and $y = 0.26$

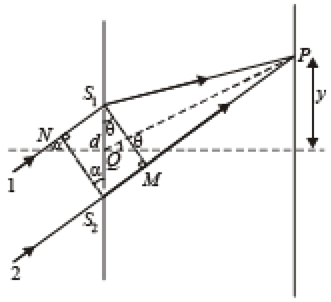
$m = -1$, $\sin\theta = -\dfrac{3}{4}$ and $y = -1.13$ m

$m = +2,\ \sin\theta = \frac{3}{4} \quad \therefore \quad y = +1.13\,\text{m}$

$m = 0,\ \sin\theta = -\frac{1}{4} \quad \therefore \quad y = -0.26\,\text{m}$

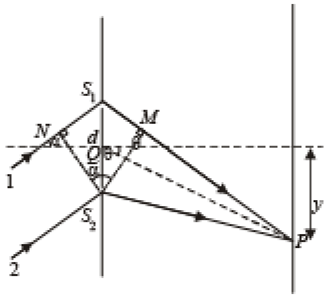
(b) WHEN THE INCIDENT BEAM MAKES AN ANGLE OF 30° WITH X-AXIS

Two cases arise as shown by the following two figures.



Path difference between ray 1 and 2 reaching P = $S_2M - NS_1$

$\therefore\ \Delta x_1 = d \sin\theta - d \sin\alpha$ (Case 1)



Path difference between ray 1 and 2 reaching P = $NS_1 + S_1M$

$\Delta x_2 = d \sin\alpha + d \sin\theta$ (Case 2)

Position of Central maxima : Path difference should be zero. Therefore

$\Delta x_1 = 0$ or $\Delta x_2 = 0$

$\Rightarrow d \sin\alpha = d \sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2} \qquad [\because \alpha = 30°]$

For first minima; $d\sin\theta + d\sin\alpha = \frac{\lambda}{2}$

$$\Rightarrow \quad d\sin\theta = \frac{\lambda}{2} + d\sin\alpha$$

$$\therefore \quad \sin\theta = \frac{\lambda}{2d} + \sin\alpha = \frac{0.5}{2\times 1} + \sin 30^\circ = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

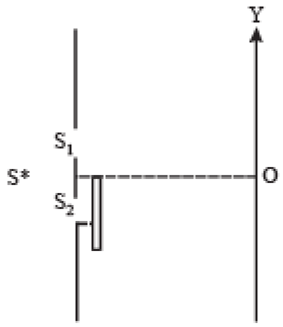
From equation (i), y = 1.15m

For first minima on the other side

$$d\sin\alpha + d\sin\theta = \frac{\lambda}{2} \quad \Rightarrow \quad \sin\theta = \frac{-1}{4}$$

$$\therefore \quad \text{From (i)}, \quad y = -0.26\text{m}$$

Q.26. The Young's double slit experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 mm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown in Figure. (1999 - 10 Marks)



(a) Find the location of the central maximum (bright fringe with zero path difference) on the y – axis.

(b) Find the light inten sity at poin t O relative to th e maximum fringe intensity.

(c) Now, if 600 nm light is replaced by white light of range 400 to 700 nm, find the wavelengths of the light that form maxima exactly at point O.

[All wavelengths in this problem are for the given medium of refractive index 4/3. Ignore dispersion]

Ans. (a) 4.33×10^{-3} m

(b) 0.75

(c) 0.65×10^{-6} m, 0.433×10^{-6} m

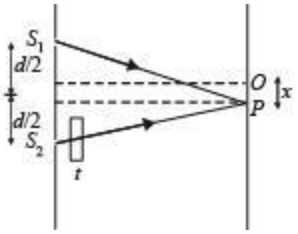
Solution. (a) Let the central maxima is obtained at a distance x below O. [This is

because a glass sheet is present in front of S_2 which increases its path length to the screen.

Therefore the path length of ray from S_1 to the screen should also increase].

Here,

$$\Rightarrow \frac{xd}{D} = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$



$$\Rightarrow x = \left(\frac{\mu_g}{\mu_m} - 1\right)t \times \frac{D}{d} = \left(\frac{1.5}{4/3} - 1\right) \times \frac{(10.4 \times 10^{-6})(1.5)}{0.45 \times 10^{-3}}$$
$$= 4.33 \times 10^{-3} \text{ m}$$

(b) For O, path difference $= \left(\frac{\mu_g}{\mu_m} - 1\right)t$

∴ Phase difference

$$\phi = \frac{2\pi}{\lambda}\left(\frac{\mu_g}{\mu_m} - 1\right)t = \frac{2 \times 3.14}{6 \times 10^{-7}}\left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{-6})$$
$$= 6.8 \text{ rad}$$

We know that $I = I_0 \cos^2\frac{\phi}{2}$ ∴ $\frac{I}{I_0} = \cos^2(6.8) = 0.75$

(c) For maximum at O

Again path difference $= \left(\frac{\mu_g}{\mu_m} - 1\right) t$

We know that for maxima, path difference = nλ

$$\therefore \quad n\lambda = \left(\frac{\mu_g}{\mu_m} - 1\right) t$$

$$\Rightarrow \lambda = \left(\frac{\mu_g}{\mu_m} - 1\right)\frac{t}{n} = \left(\frac{1.5}{4/3} - 1\right)\frac{10.4 \times 10^{-6}}{n}$$

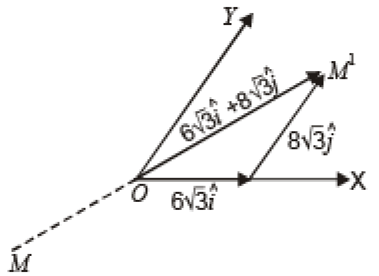
$$= \frac{1.3 \times 10^{-6}\,\text{m}}{n}$$

Putting different values of n for find the wave length in the range of 0.4×10^{-6} m to 0.7×10^{-6} m we get $\lambda = 0.65 \times 10^{-6}$ m and 0.433×10^{-6} m

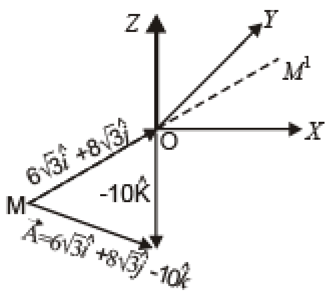
Q.27. The x – y plane is the boundary between two transparent media. Medium –1 with z ≥0 has a refractive index √2 and medium –2 with z ≤ 0 has a refractive index √3 . A ray of light in medium –1 given by the vector $A = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}$ –10 k is incident on the plane of separation. Find the unit vector in the direction of the refracted ray in medium –2. (1999 - 10 Marks)

Ans. $\frac{1}{5\sqrt{2}}\left[3\hat{i} + 4\hat{j} - 5\hat{k}\right]$

Solution.



(Fig. 1)



(Fig. 2)

Figure 1 shows vector $OM' = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}$

Figure 2 shows vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$

The perpendicular to line MOM' is Z-axis which has a unit vector of $\hat{k}$.

Angle between vector $\overrightarrow{MP}$ and $\overrightarrow{OP}$ can be found by dot product.

$$\overrightarrow{MP} \cdot \overrightarrow{OP} = (MP)\,(OP)\cos i$$

$$\frac{\left(6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}\right).(-\hat{k})}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + (-10)^2} + \sqrt{(-1)^2}} = \cos i$$

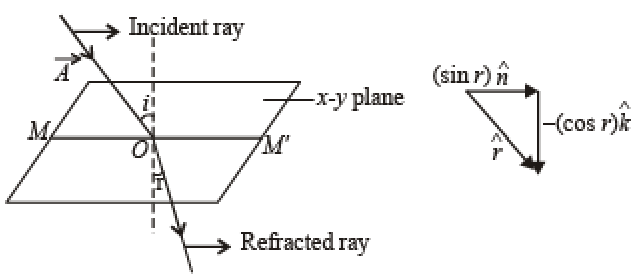
$\Rightarrow$ i = 60°

Unit vector in the direction of MOM' from fig. (1) is

$$\hat{n} = \frac{6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}}{[(6\sqrt{3})^2 + (8\sqrt{3})^2]^{1/2}}, \quad \hat{n} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

To find the angle of refraction, we use Snell's law

$$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin r} \Rightarrow r = 45°$$



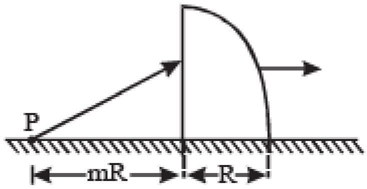
Now, $\hat{r} = (\sin r)\hat{n} - (\cos r)\hat{k}$

$$= (\sin 45°)\left[\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right] - (\cos 45°)\hat{k}$$

$$= \frac{1}{5\sqrt{2}}[3\hat{i} + 4\hat{j} - 5\hat{k}]$$

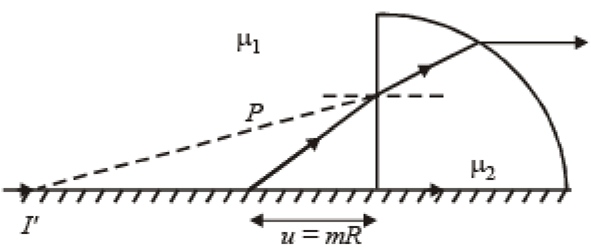
Q.28. A quarter cylinder of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance of mR from it. Find the value of m for which a ray from P will emerge parallel to the table as shown in

Figure. (1999 - 5 Marks)



Ans. 4/3

Solution. First of all, we consider the refraction at plane surface. Here the image of P will form at I' after refraction from I surface.



For plane surface :

Object distance u = – mR

Radius of curvature of the plane surface = ∞

The ray is coming from air and incident on the glas.

Here $\mu_1 = 1$, $\mu_2 = 1.5$.

Apply $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}; \frac{\mu_2}{v} = \frac{\mu_1}{u}$ (as $R = \infty$)

$\therefore$ Image distance $v = \frac{\mu_1}{\mu_2} u = \frac{1.5}{1.0}(-mR) = -1.5\,mR$

Now we consider refraction at the curved surface.

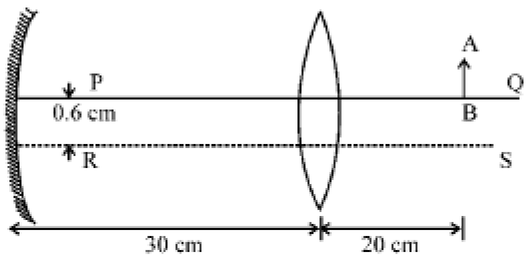
Object distance, u = – (1.5 mR + R)

Here, $\mu_2 = 1$, $\mu_1 = 1.5$, Image distance, v = ∞,

Radius of curvature = – R

Here, $\frac{1}{\infty} + \frac{1.5}{(1.5m+1)R} = \frac{1-1.5}{-R} \quad \therefore \ m = \frac{4}{3}$

Q.29. A convex lens of focal length 15 cm and a cancave mirror of focal length 30 cm are kept with their optic axes PQ and RS parallel but separated in vertical direction by 0.6 cm as shown. The distance between the lens and mirror is 30 cm. An upright object AB of height 1.2 cm is placed on the optic axis PQ of the lens at a distance of 20 cm from the lens. If A'B' is the image after refraction from the lens and reflection from the mirror, find the distance of A'B' from the pole of the mirror and obtain its magnification. Also locate position of A' and B' with respect to the optic axis RS. (2000 – 6 Marks)

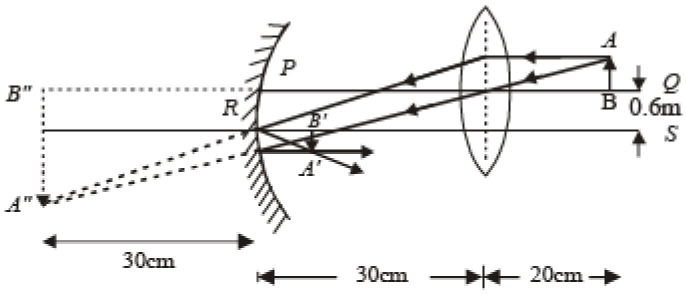


(b) A glass plate of refractive index 1.5 is coated with a thin layer of thickness t and refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surface of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If l = 648 nm, obtain the least value of t for which the rays interfere constructively. (2000 - 4 Marks)

Ans. (a) 15 cm, 1/2

(b) 90 nm

Solution. (a) For the lens



$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_\ell} \Rightarrow \frac{1}{v}-\frac{1}{-20}=\frac{1}{15}$$

$$\Rightarrow \frac{1}{v}=\frac{1}{15}-\frac{1}{20}=\frac{1}{60} \Rightarrow v=60\text{ cm}, \quad m=\frac{v}{u}=\frac{60}{-20}=-3$$

The image is formed to the left of the lens, real, inverted and three times the actual size (3.6 cm in height below PQ).

For the mirror,

$$\frac{1}{v'}+\frac{1}{u'}=\frac{1}{f_m} \Rightarrow \frac{1}{v'}=\frac{1}{-30}-\frac{1}{30}=-\frac{2}{30}$$

$\Rightarrow$ v' = – 15 cm

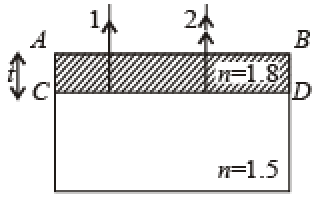
$$m=-\frac{v'}{u'}=-\frac{-15}{30}=\frac{1}{2}$$

$$\text{size of image}=\frac{1}{2}\times 3.6=1.8\text{ cm.}$$

This image will be inverted w.r.t. the original image and its position will be 0.3 cm above RS and 1.5 cm below RS. The position of the image is 15 cm to the right of the mirror.

(b) The path difference between the two rays reflected from the upper surface AB (shown by ray 1, single arrow upwards) and lower surface CD (shown by ray 2 double arrow pointing upwards) is

$$\Delta x = {}_{m}n \times 2t + \frac{\lambda}{2}$$



Here λ/2 is the path difference as the ray 1 suffer reflection from a denser medium on surface AB

We known that for constructive interference

Path difference = mλ where m is 1, 2,....

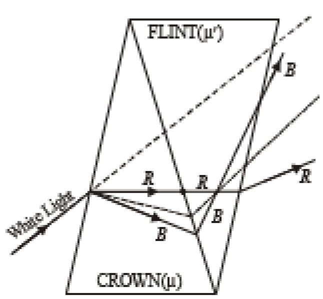
$$\therefore \quad {}_{m}n \times 2t + \frac{\lambda}{2} = m\lambda \;\Rightarrow\; 2\,{}_{m}nt = \left(m - \frac{1}{2}\right)\lambda$$

$$\text{when } m = 1,\ t = \frac{\lambda}{4\,{}_{m}n} = \frac{648}{4 \times 1.8} = 90\,\text{nm.}$$

Q.30. The refractive indices of the crown glass for blue and red lights are 1.51 and 1.49 respectively and those of flint glass are 1.77 and 1.73 respectively. An isosceles prism of angle 6o is made of crown glass. A beam of white light is incident at a small angle on this prism. The other flint glass isosceles prism is combined with the crown glass prism such that there is no deviation of the incident light. Determine the angle of the flint glass prism. Calculate the net dispersion of the combined system. (2001 - 5 Marks)

Ans. 4°, – 0.04°

Solution. For no deviation condition



$$A' = \left[\frac{\mu - 1}{\mu' - 1}\right] A$$

$\Rightarrow \quad A' = \frac{1.5-1}{1.75-1} \times 6° = 4°$

Now, the angular dispersion produced by crown glass prism $\delta_b - \delta_r = A(\mu_b - \mu_r)$

Also the angular dispersion produced by flint glass prism

$\delta_b' - \delta_r' = A'(\mu_b' - \mu_r')$

∴ Net deviation in blue light

$\delta b = (\mu_{b1} - 1) A1 - (\mu_{b2} - 1)A_2$

$= (1.51 - 1)\, 6° - (1.77 - 1)\, 4° = -0.02°$

Similarly Net deviation of red light

$\delta r = (\mu_{r1} - 1) A_1 - (\mu_{r2} - 1)A_2$

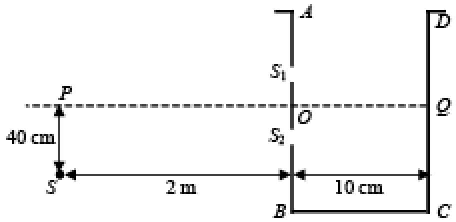
$= (1.49 - 1)\, 6° - (1.73 - 1)\, 4° = 0.02$

∴ Net dispersion $= \delta_b - \delta_r = -0.04°$

∴ The magnitude of the net angular dispersion = 0.04

Q.31. A vessel ABCD of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel, to illuminate the slits as shown in the figure below. Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ. Now, a liquid is poured into the vessel and filled upto OQ. The central bright fringe is found to be at Q.

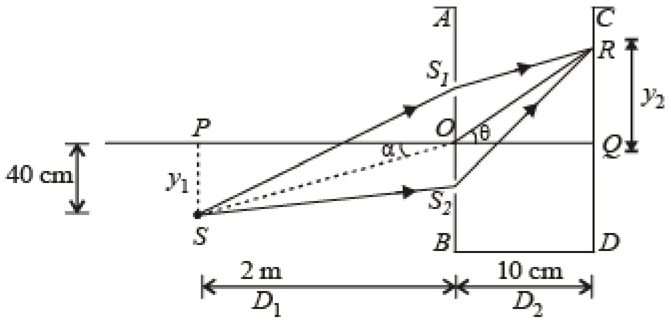
Calculate the refractive index of the liquid. (2001-5 Marks)



Ans. 2 cm, 1.0016

Solution. (i) O is the middle point of two slits S_1 and S_2.

$S_1S_2 = d = 0.8$ mm



$$\tan\alpha = \frac{y_1}{D_1} = \frac{40}{200} = \frac{1}{5}$$

$$\therefore \quad \sin\alpha = \frac{1}{\sqrt{26}} = \frac{1}{5.1} \approx \frac{1}{1.5} \approx \tan\alpha$$

or $\Delta X_1 = d \sin\alpha$ $= (0.8 \text{ mm})\left(\frac{1}{5}\right) = 0.16 \text{ mm} \quad \ldots \text{(i)}$

Let R denotes the position of central bright fringe. Net path difference will be zero.

Now $\Delta X_2 = S_2R - S_1R$ or $\Delta X_2 = \ d\sin\theta$ ... (ii)

For central bright fringe

$\Delta X_2 - \Delta X_1 = 0$ or $\quad d\sin\theta - \Delta X1 = 0$

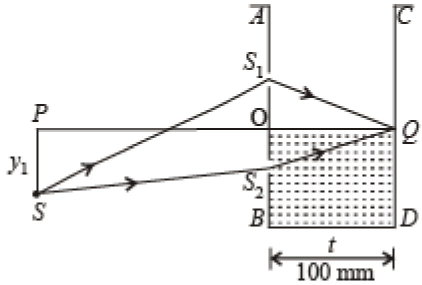
or $= d\sin\theta = \Delta X_1 = 0.16$ mm or $(0.8)\sin\theta = 0.16$ or $\sin\theta = \frac{0.16}{0.8} = \frac{1}{5}$

$\therefore \tan\theta = \frac{1}{\sqrt{24}} = \frac{1}{4.9} \approx \frac{1}{5} = \sin\theta$ $\therefore \tan\theta = \frac{y_2}{D_2}$

or $\frac{1}{5} = \frac{y_2}{D_2}$ or $y_2 = \frac{D_2}{5} = \frac{10}{5} = 2$ cm

Hence position of central bright fringe is 2 cm above point Q on side CD.

(ii) μ of liquid poured if central fringe is at Q:



The liquid is poured into vessel upto OQ.

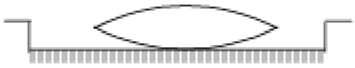
The central bright fringe is formed at Q.

For central bright fringe net path difference = 0.

$(\mu - 1)t = \Delta X_1$ or $(\mu - 1)(100) = 0.16$

or $\mu - 1 = 0.0016$ or $\mu = 1.0016$

Q.32. A thin biconvex lens of refractive index 3/2 is placed on a horizontal plane mirror as shown in the figure. The space between the lens and the mirror is then filled with water of refractive index 4/3. It is found that when a point object is placed 15 cm above the lens on its principal axis, the object coincides with its own image. On repeating with another liquid, the object and the image again coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid. (2001-5 Marks)



Ans. 1.6

Solution. The lens maker formula is

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

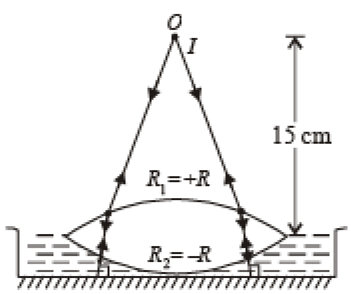
When the space between the lens and the mirror is filled with water, a system of two lenses is formed.

(i) a glass lens

(ii) a plano concave water lens

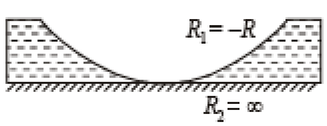
For glass lens Here R_1 = + R and R_2 = – R

$$\frac{1}{f_\ell} = (1.5 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{1}{R}$$



For water lens

$$\frac{1}{f_w} = (1.33 - 1)\left(\frac{1}{-R} - \frac{1}{-\infty}\right) = \frac{-0.33}{R}$$



The focal length of the combination of two lenses will be

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{R} - \frac{0.33}{R} = \frac{0.67}{R} \qquad \text{...(i)}$$

A convex lens placed on a plane mirror behaves like a concave mirror. The image is formed at the object itself if the object is placed at centre of curvature of concave mirror.

After refraction through lens, the rays fall on the plane mirror normally and retrace their path to form image at the object itself.

∴ Focal length of system (f) = 15 cm ... (ii)

From (i) and (ii)

$$\frac{1}{15} = \frac{0.67}{R} \Rightarrow R = 10.05 \text{ cm}$$

The same situation is repeated with two differences

(a) The object and image distance are now 25 cm and

(b) In place of water there is a new liquid of refraction index μ

$$\text{Again } \frac{1}{f_\ell} = \frac{1}{R} \text{ and } \frac{1}{f'} = \frac{-(\mu - 1)}{R}$$

where f ' is the focal

length of new liquid lens.

∴ New combined lens,

$$\frac{1}{F} = \frac{1}{f_\ell} + \frac{1}{f'} = \frac{1}{R} - \frac{+(\mu - 1)}{R} = \frac{1 - \mu + 1}{R} = \frac{2 - \mu}{R} \quad \text{.... (i)}$$

For new combined lens,

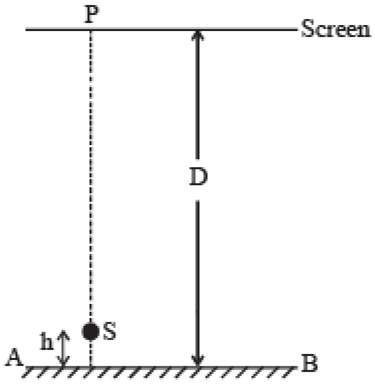
$$\therefore \quad \frac{1}{F} = \frac{1}{25} \quad \text{... (ii)}$$

From (i) and (ii)

$$\frac{2 - \mu}{10.02} = \frac{1}{25} \quad \therefore \quad \mu = 1.6$$

Q.33. A point source S emitting light of wavelength 600 nm is placed at a very small height h above a flat reflecting surface AB (see figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are

observed on a screen placed parallel to the reflecting surface at a very large distance D from it. (2002 - 5 Marks)



(a) What is the shape of the interference fringes on the screen?

(b) Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P (shown in the figure).

(c) If the intensity at point P corresponds to a maximum, calculate the minimum distance through which the reflecting surface AB should be shifted so that the intensity at P again becomes maximum.

Ans. (a) circular

(b) 1/16

(c) 300 nm

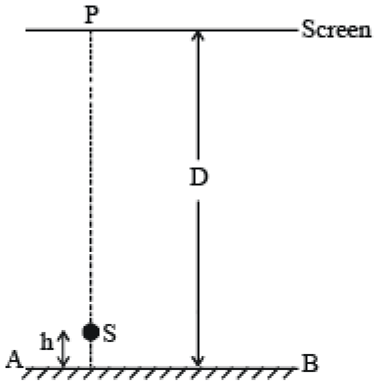
Solution. (a) Because S is a point source, fringes will be circular.

(b) $$\frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{I} - \sqrt{0.36\,I}}{\sqrt{I} + \sqrt{0.36\,I}}\right)^2 = \left(\frac{0.4}{1.6}\right)^2 = \frac{1}{16}$$

[∵ If intensity of light falling on P directly from S is I, then the intensity of light falling at P after reflection from AB is 0.36 I]

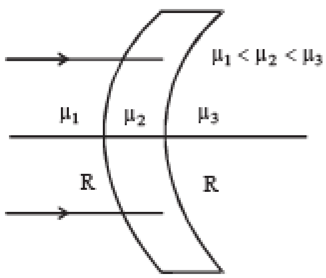
(c) For maximum at P, path difference = $n\lambda$

If AB is shifted by a distance x, it will cause an additional path difference of 2x.



$2x = \lambda$ (for minimum value of x) $\Rightarrow x = \lambda/2 = 300$ nm

Q.34. Find the focal length of the lens shown in the figure. The radii of curvature of both the surfaces are equal to R. (2003 - 2 Marks)



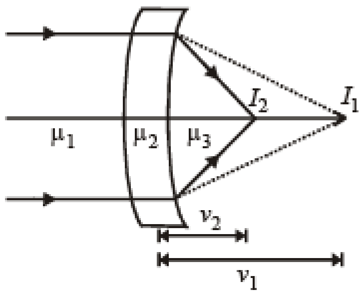
Ans. $\frac{\mu_3 R}{\mu_3 - \mu_1}$

Solution. For an object placed at infinity the image after first refraction will be formed at a distance v_1

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{-\infty} = \frac{\mu_2 - \mu_1}{+R} \quad \text{... (i)}$$

Image afte second refraction will be formed at a distance v_2

$$\frac{\mu_3}{v_2} - \frac{\mu_2}{v_1} = \frac{\mu_3 - \mu_2}{+R} \quad \text{... (ii)}$$



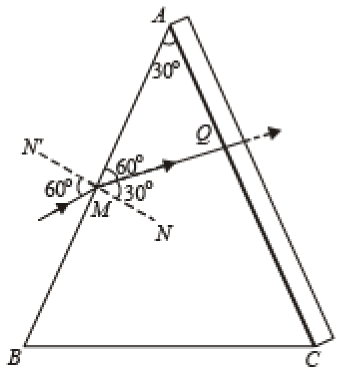
Adding (i) and (ii),

$$\frac{\mu_3}{v_2} - \frac{\mu_3 - \mu_1}{R} \Rightarrow v_2 = \frac{\mu_3 R}{\mu_3 - \mu_1}$$

Final image is formed at the focus when incident rays are parallel.

Therefore, focal length will be $\frac{\mu_3 R}{\mu_3 - \mu_1}$

Q.35. Shown in the figure is a prism of angle 30° and refractive index $\mu_p = \sqrt{3}$. Face AC of the prism is covered with a thin film of refractive index $\mu_f = 2.2$. A monochromatic light of wavelength $\lambda = 550$ nm fall on the face AB at an angle of incidence of 60°. (2003 - 4 Marks)



Calculate

(a) angle of emergence.

(b) minimum value of thickness t so that intensity of emergent ray is maximum.

Ans. (a) zero

(b) 125 nm

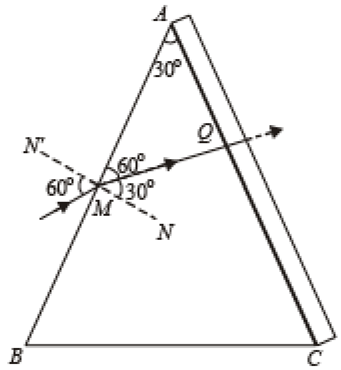
Solution. (a) Using Snell's law at surface AB

$$\mu_{air} \sin 60° = \mu_p \sin r \Rightarrow \frac{\sqrt{3}}{2} = \sqrt{3} \sin r \Rightarrow r = 30°$$

Now, NN' is the normal to surface AB.

∴∠ AMN = 90°

But ∠ QMN = 30° ⇒ ∠AMQ = 60°



In ΔAMQ

∠ AQM = 180° – (60° + 30°) = 90°

The refracted ray inside the prism hits the other face at 90°; hence deviation produced by this face is zero and hence angle of emergence is zero.

(b) Multiple reflections occur in the film for minimum thickness.

The intensity of emergent ray will be maximum if transmitted waves undergo constructive interference.

∴ For minimum thickness,

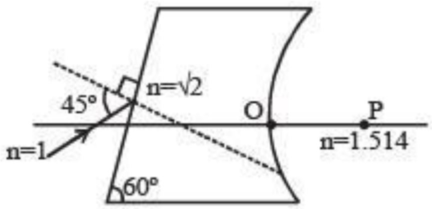
⇒ $\Delta x = \lambda$

⇒ $\Delta x = 2mt = \lambda$,

where t = thickness $\Rightarrow t = \frac{\lambda}{2\mu} = 125\,\text{nm}$

Q.36. A ray is incident on a medium consisting of two boundaries, one plane and other curved as shown in the figure. The plane surface makes an angle 60º with horizontal and curved surface has radius of curvature 0.4 m. The refractive indices of the medium and its environment are shown in the figure.

If after refraction at both the surfaces the ray meets principle axis at P, find OP. (2004 - 2 Marks)



Ans. 6.056 m

Solution. Use Snell's law

$n_1 \sin i = n_2 \sin r$

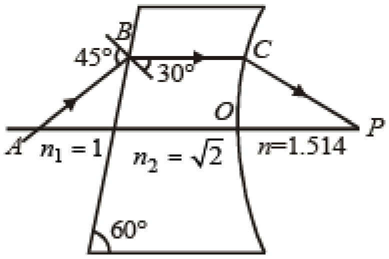
Here, $n_1 = 1, n_2 = \sqrt{2}, i = 45°, r = ?$

$\Rightarrow \sin r = \frac{1 \times \sin 45°}{\sqrt{2}} = \frac{1}{2} \Rightarrow r = 30°$

The angle made by refracted ray at B with normal is 30°.

∴ Angle made by the first surface with refracted ray BC is 60°.

Hence the refracted ray at B is parallel to horizontal arrow.



∴ For refraction at spherical surface, u = ∞

Now, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

$\Rightarrow \frac{1.514}{v} = \frac{1.514 - 1.414}{0.4}$ or $v = 6.056\text{ m}$

$\therefore \quad OP = 6.056\text{ m}$

Q.37. In YDSE a light containing two wavelengths 500 nm and 700 nm are used. Find the minimum distance where maxima of two wavelengths coincide. Given D/d = 10^3, where D is the distance between the slits and the screen and d is the distance between the slits. (2004 - 4 Marks)

Ans. 3.5 mm

Solution. At the place where maxima for both the wavelengths coincide, y will be same for both the maxima, i.e.,

$$\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d} \Rightarrow \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{700}{500} = \frac{7}{5}$$

Minimum integral value of n_2 is 5.

∴ Minimum distance of maxima of the two wavelengths from central fringe

$$= \frac{n_2\lambda_2 D}{d} = 5 \times 700 \times 10^{-9} \times 10^3 = 3.5\text{ mm.}$$

Q.38. An object is moving with velocity 0.01 m/s towards a convex lens of focal length 0.3 m. Find the magnitude of rate of separation of image from the lens when the object is at a distance of 0.4 m from the lens. Also calculate the magnitude of the rate of change of the lateral magnification. (2004 - 4 Marks)

Ans. 0.09 m/sec, 0.35 s^{-1}

Solution. f = 0.3 m, u = – 0.4 m

Using lens formula

$$\frac{1}{v} - \frac{1}{-0.4} = \frac{1}{0.3} \Rightarrow v = 1.2\text{m}$$

Now we have $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, differentiating w.r.t. t

we have $-\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0$ given $\frac{du}{dt} = 0.01$ m/s

$$\Rightarrow \left(\frac{dv}{dt}\right) = \frac{(1.20)^2}{(0.4)^2} \times 0.01 = 0.09 \text{ m/s}$$

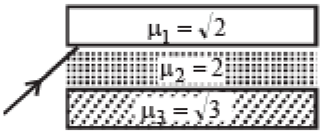
So, rate of separation of the image (w.r.t. the lens) = 0.09 m/s

$$\text{Now, } m = \frac{v}{u} \Rightarrow \frac{dm}{dt} = \frac{u\frac{dv}{dt} - v\frac{du}{dt}}{u^2}$$

$$= \frac{-(0.4)(0.09) - (1.2)(0.01)}{(0.4)^2} = -0.35 \text{ s}^{-1}$$

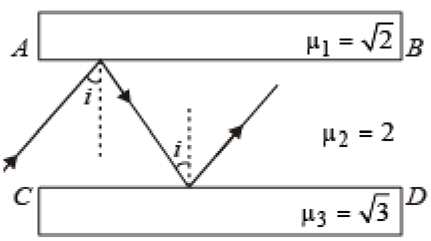
Magnitude of rate of change of lateral magnification = 0.35 s^{-1}.

Q.39. What will be the minimum angle of incidence such that the total internal reflection occurs on both the surfaces? (2005 - 2 Marks)



Ans. 60°

Solution. For total internal reflection on interface AB



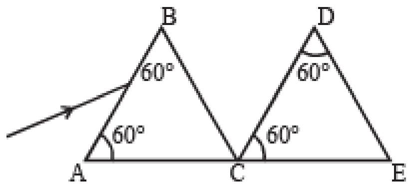
$$\sin i = \frac{1}{{}_1^2\mu} = \frac{{}_1\mu}{{}_2\mu} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \Rightarrow i = 45^\circ$$

For total internal reflection in interface CD.

$$\sin i = \frac{1}{\frac{3}{2}\mu} = \frac{3\mu}{2\mu} = \frac{\sqrt{3}}{2} \Rightarrow i = 60°$$

⇒ The minimum angle for total internal reflection for both the interface is 60°.

Q.40. Two identical prisms of refractive index √3 are kept as shown in the figure. A light ray strikes the first prism at face AB. Find, (2005 - 4 Marks)



(a) the angle of incidence, so that the emergent ray from the first prism has minimum deviation.

(b) through what angle the prism DCE should be rotated about C so that the final emergent ray also has minimum deviation.

Ans. (a) 60º,

(b) 60º, anticlockwise

Solution. (a) For minimum deviation of emergent ray from the first prism. MN is parallel to AC

∴ ∠BMN = 60°

⇒ ∠r = 30°

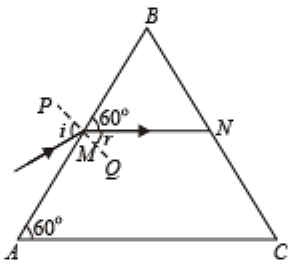
Applying Snell's law at M

$$\mu = \frac{\sin i}{\sin r}$$

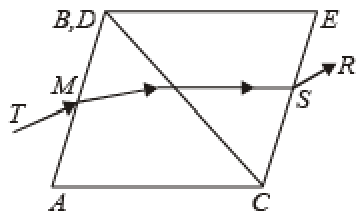
$$\sin i = \mu \sin r$$

$$\sin i = \sqrt{3} \times \sin 30^\circ = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad i = 60^\circ$$



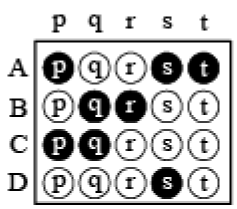
(b) When the prism DCE is rotated about C in anticlockwise direction by 60°, as shown in the figure, then the final emergent ray SR becomes parallel to the incident ray TM. Thus, the angle of deviation becomes zero.



Match the Following

DIRECTIONS (Q. No. 1-4) : Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in ColumnII are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



Q.1. A simple telescope used to view distant objects has eyepiece and objective lens of focal lengths fe and fo, respectively. Then (2006 - 6M)

Column I	**Column II**
(A) Intensity of light received by lens	(p) Radius of aperture
(B) Angular magnification	(q) Dispersion of lens
(C) Length of telescope	(r) Focal length of objective lens and eyepiece lens
(D) Sharpness of image	(s) Spherical aberration

Ans. A-p; B-r; C-r; D-p, q, s

Solution. (A) → (p).

More the radius of aperture more is the amount of light entering the telescope.

(B) → (q).

$M = \frac{f_0}{f_e}$

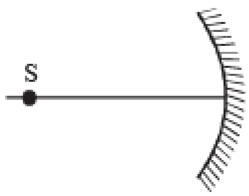
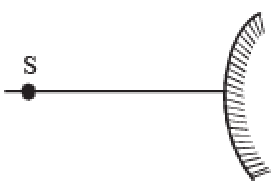
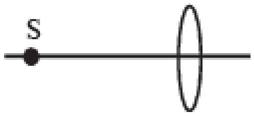
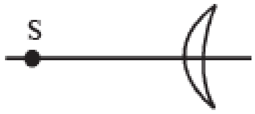
(C) → (r).

$L = f_0 + f_e$

(D) → (p), (q), (r).

Depends on dispersion of lens, spherical aberration and radius of aperture.

Q.2. An optical component and an object S placed along its optic axis are given in Column I. The distance between the object and the component can be varied. The properties of images are given in Column II. Match all the properties of images from Column II with the appropriate components given in Column I. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

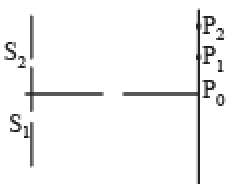
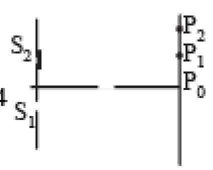
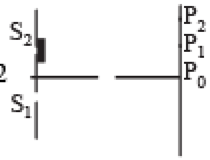
Column I	Column II
(A) 	(p) real image
(B) 	(q) virtual image
(C) 	(r) magnified image
(D) 	(s) image at infinity

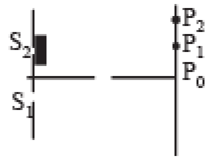
Ans. A-p, q, r, s; B-q; C-p, q, r, s; D-p, q, r, s

Solution. A-p, q, r, s l

- When th e object is at in finity, a real, inver ted and diminished image is formed at the focus of the concave mirror.
- As the object is brought closer to the mirror, the image moves farther, remains real and inverted and increases in size (but still it is diminished as compared to the object) l When the object is at C, a real, inverted and same size image is formed at C.
- When the object is brought still closer, a real, inverted and magnified image is formed beyond C.
- When the object is at focus (F), the image is highly magnified, real and inverted and formed at infinity.
- When the object is placed between pole and focus, a virtual, erect and magnified image is formed behind the mirror.

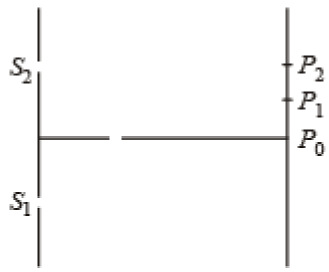
Q.3. Column-I shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \lambda/4$ and $S_1P_2 - S_2P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by d (P) and the intensity by δ(P). Match each situation given in Column-I with the statetment(s) in Column-II valid for that situation.

Column-I	**Column-II**
(A) 	(p) $\delta(P_0) = 0$
(B) $(\mu-1)t=\lambda/4$ 	(q) $\delta(P_1) = 0$
(C) $(\mu-1)t=\lambda/2$ 	(r) $I(P_1) = 0$

(D) $(\mu - 1)t = 3\lambda/4$ 	(s) $I(P_0) > I(P_1)$
	(t) $I(P_2) > I(P_1)$

Ans. A-p, s; B-q; C-t; D-r, s, t

Solution.



For path difference $\lambda/4$, phase difference is $\pi/2$.

For path difference $\lambda/3$, phase difference is $2\pi/3$.

Here, $S_1P_0 - S_2P_0 = 0$

$\therefore \delta(P_0) = 0$

Therefore, (p) matches with (A).

The path difference for P_1 and P_2 will not be zero. The intensities at P_0 is maximum.

$$I(P_0) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos 0°$$

$$= (\sqrt{I_1} + \sqrt{I_2})^2 = (I_0 + I_0)^2 = 4I_0$$

$$I(P_1) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\frac{\pi}{2}$$

$$= I_1 + I_2 = I_0 + I_0 = 2I_0$$

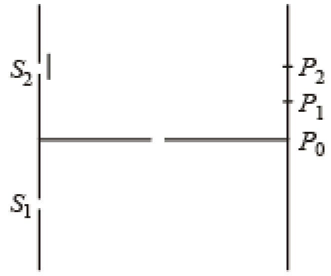
$I(P_2) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(2\pi/3)$

$= I_1 + I_2 - \sqrt{I_1}\sqrt{I_2} = I_0 + I_0 - I_0 = I_0$

$\therefore \quad I(P_0) > I(P_1)$

Therefore, (s) matches with (A).

(B)



$\delta P_0 = \frac{\lambda}{4}, \; \delta P_1 = 0, \; \delta P_2 = \frac{\lambda}{12}$

$I(P_0) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\pi/2$

$= I_1 + I_2 = I_0 + I_0 = 2I_0$

$I(P_1) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} = 4I_0$

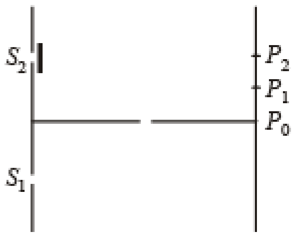
$I(P_2) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\pi/6$

$= I_1 + I_2 + \sqrt{3}\sqrt{I_1}\sqrt{I_2}$

$= I_0 + I_0 + \sqrt{3}I_0 \quad = (2+\sqrt{3})I_0$

Therefore, q match with (B)

(C)



Here $\delta(P_0) = -\lambda/2;\ \delta(P_1) = -\lambda/4,\ \delta(P_2) = -\lambda/6$

$$I(P_0) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(-\pi)$$

$$= I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2} = I_0 + I_0 - 2I_0 = 0$$

$$I(P_1) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(-\pi/2)$$

$$= I_1 + I_2 = I_0 + I_0 = 2I_0$$

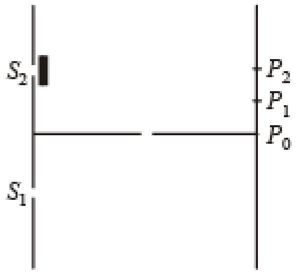
$$I(P_2) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\left(-\frac{\pi}{3}\right)$$

$$= I_1 + I_2 + \sqrt{I_1}\sqrt{I_2} = I_0 + I_0 + I_0 = 3I_0$$

$\therefore$ I (P_2) > I (P_1)

(t) matches (C).

(D)



Here $\delta P_0 = 3\lambda/4; \delta P_1 = -\lambda/2; \delta P_2 = -5\lambda/12$

$$I(P_0) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\left(\frac{-3\pi}{2}\right)$$

$$= I_1 + I_2 = I_0 + I_0 = 2I_0$$

$$I(P_1) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(-\pi)$$

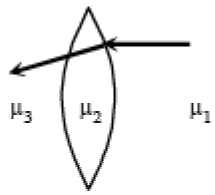
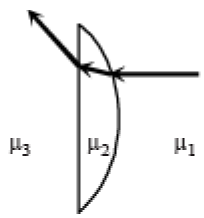
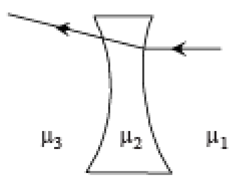
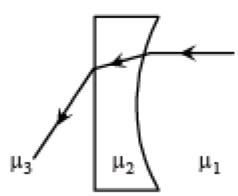
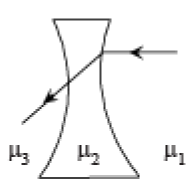
$$= I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2} = I_0 + I_0 - 2\sqrt{I_0}\sqrt{I_0} = 0$$

$$I(P_2) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos[-5\pi/6]$$

$= I_1 + I_2 - \sqrt{3}\sqrt{I_1}\sqrt{I_2} = \left(2-\sqrt{3}\right) I_0$

(r), (s), (t) matches (D).

Q.4. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in Column II. A ray traversing these media is also shown in the figures. In Column I different relationships between μ_1, μ_2, and μ_3 are given. Match them to the ray diagrams shown in Column II. (2010)

Column I	**Column II**
(A) $\mu_1 < \mu_2$	(p) 
(B) $\mu_1 > \mu_2$	(q) 
(C) $\mu_2 = \mu_3$	(r) 
(D) $\mu_2 > \mu_3$	(s) 
	(t) 

Ans. A-p, r; B-q, s, t; C-p, r, t; D-q, s

Solution. (a) When $\mu_1 < \mu_2$, the ray of light while entering the lens will bend towards the normal. Therefore p, r are the correct options

(B) When $\mu_1 > \mu_2$, the ray of light while entering the lens will bend away from the normal. Therefore q,s,t are the correct options.

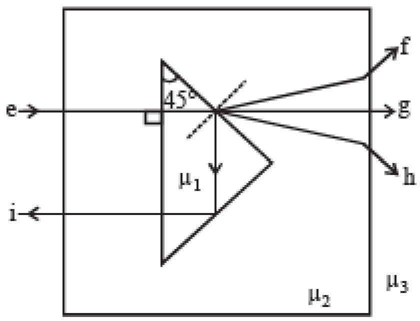
(C) When $\mu_2 = \mu_3$, the ray of light while coming out from the lens does not deviate from its path. Therefore p,r,t are the correct option.

(D) $\mu_2 > \mu_3$, the ray of light coming out of the lens deviates away from the normal. Therefore q,s are the correct options.

DIRECTION (Q. No. 5 & 6) Following question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

Q.5. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light 'e' enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths 'ef', 'eg', 'eh' or 'ei'.

Match the paths in List I with conditions of refractive indices in List II and select the correct answer using the codes given below the lists: (JEE Adv. 2013)



List I	**List II**
P. $e \rightarrow f$	1. $\mu_1 > 2\mu_2$
Q. $e \rightarrow g$	2. $\mu_2 > \mu_1$ and $\mu_2 > \mu_3$

R. $e \rightarrow h$	3. $\mu_1 = \mu_2$
S. $e \rightarrow i$	4. $\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$

Codes:

	P	Q	R	S
(a)	2	3	1	4
(b)	1	2	4	3
(c)	4	1	2	3
(d)	2	3	4	1

Ans. (d)

Solution. $e \rightarrow f$. For the ray to bend towards the normal at the prism surface $\mu_2 > \mu_1$. The ray then moves away from the normal when it emerges out of the rectangular block. Therefore $\mu_2 > \mu_3$.

$e \rightarrow g$. As there is no deviation of the ray as it emerges out of the prism, $\mu_2 = \mu_1$.

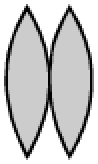
$e \rightarrow h$. As the ray emerges out of prism, it moves away from the normal. Therefore $\mu_2 < \mu_1$. As the ray moves away from the normal as it emerges out of the rectangular block, therefore $\mu_2 > \mu_3$.

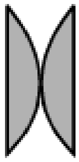
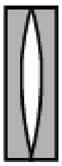
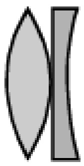
$e \rightarrow i$. At the prism surface, total internal reflection has taken place. For this

$\sin 45° > \frac{h_2}{h_1}$

$\therefore$ m1 > $\sqrt{2}\,\mu_2$. (d) is the correct option.

Q.6. Four combinations of two thin lenses are given in List-I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in List-I with their focal length in List-II and select the correct answer using the code given below the lists. (JEE Adv. 2014)

List - I	**List - II**
P. 	1. 2r

Q. 	2. r/2
R. 	3. – r3. – r
S. 	4. r

Codes:

(a) P-1, Q-2, R-3, S-4

(b) P-2, Q-4, R-3, S-1

(c) P-4, Q-1, R-2, S-3

(d) P-2, Q-1, R-3, S-4

Ans. (b)

Solution. For (P) $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$= (1.5 - 1)\left[\frac{2}{r}\right] = \frac{1}{r} \Rightarrow f = r$

For the combination

$\therefore \quad F = \frac{r}{2}$

For (Q) $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$$= (1.5-1)\left[\frac{1}{\infty} - \frac{1}{-r}\right] = \frac{0.5}{r} = \frac{1}{2r}$$

∴ f = 2r

For the combination

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{2r} + \frac{1}{2r} = \frac{2}{2r} = \frac{1}{r}$$

∴ f = r

Similarly, we can either find or do not find the remaining options (b) is the correct option.

Integer Value of Ray

Q.1. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m25 to m50. The ratio $\frac{m_{25}}{m_{50}}$ is **(2010)**

Ans. 6

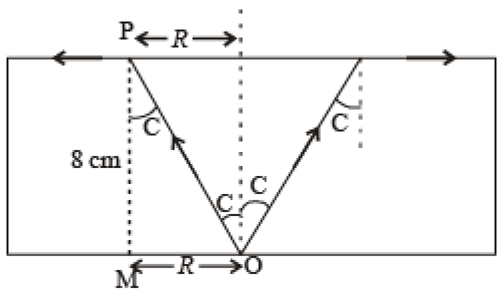
Solution. Given $f = +20\text{cm}$ Also $m = \frac{f}{f+u}$

$$\therefore \frac{m_{25}}{m_{50}} = \frac{\frac{20}{20-25}}{\frac{20}{20-50}} = \frac{-30}{-5} = 6$$

Q.2. A large glass slab (μ= 5 / 3) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R? (2010)

Ans. 6

Solution. In the figure, C represents the critical angle



$$\therefore \sin C = \frac{1}{\mu} = \frac{3}{5} \quad \therefore \tan C = \frac{3}{4}$$

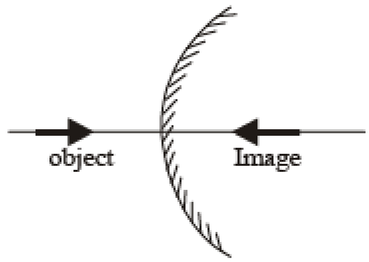
$$\text{In } \Delta \text{POM}, \tan C = \frac{\text{OM}}{\text{PM}} = \frac{R}{8}$$

$$\therefore R = \frac{3}{4} \times 8 = 6\text{cm}$$

Q.3. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from 25/3 m to 50/7 m in 30 seconds. What is the speed of the object in km per hour? (2010)

Ans. 3

Solution. Using mirror formula for first position



$u_1 = ?, v_1 = \frac{25}{3}\text{cm}, f = +10\text{cm}$

$\frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f}, \quad \frac{3}{25} + \frac{1}{u_1} = \frac{1}{10} \therefore u_1 = -50\text{ m}$

Using mirror formula for the second position

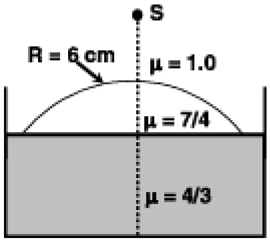
$\frac{1}{v_2} + \frac{1}{u_2} = \frac{1}{f} \Rightarrow \frac{7}{50} + \frac{1}{u_2} = \frac{1}{10} \Rightarrow \frac{1}{u_2} = \frac{1}{10} - \frac{7}{50}$

$u_2 = -25\text{m}$

Change in position of object = 25 m

$\text{Speed of object} = \frac{25}{30} \times \frac{18}{5} = 3\text{ km h}^{-1}$

Q.4. Water (with refractive index = 4/3) in a tank is 18 cm deep. Oil of refractive index 7/4 lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is (2011)

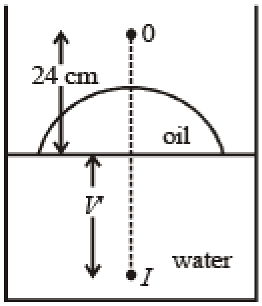


Ans. 2

Solution. For the convex spherical refracting surface of oil we apply

$$\frac{-\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$$

$$\therefore \frac{-1}{(-24)}+\frac{7/4}{v}=\frac{\frac{7}{4}-1}{6}$$



∴ v = 21 cm

For water-oil interface

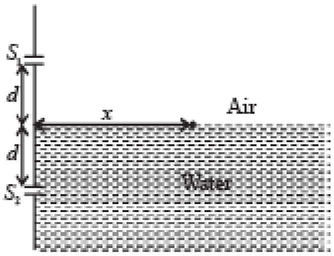
$$\frac{\frac{-7}{4}}{+21}+\frac{\frac{4}{3}}{V'}=0$$

∴ V' = 16 cm.

This is the image distance from water-oil interface.

Therefore the distance of the image from the bottom of the tank is 2 cm.

Q.5. A Young's double slit interference arrangement with slits S1 and S2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maximum on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where l is the wavelength of light in air (refractive index = 1), 2d is the separation between the slits and m is an integer. The value of p is (JEE Adv. 2015)

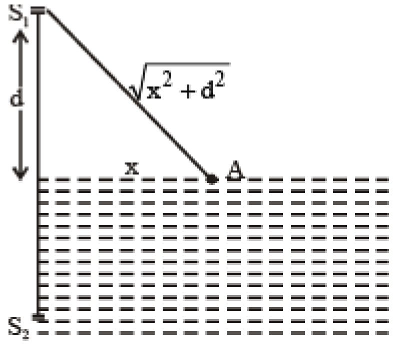


Ans. 3

Solution. For maxima

Path defference = mλ

∴ $S_2A - S_1A = m\lambda$



$$\therefore \quad \left[(n-1)\sqrt{d^2+x^2}+\sqrt{d^2+x^2}\right]-\sqrt{d^2-x^2}=m\lambda$$

$$\therefore \quad (n-1)\sqrt{(d^2+x^2)}=m\lambda$$

$$\therefore \quad \left(\frac{4}{3}-1\right)\sqrt{d^2+x^2}=m\lambda$$

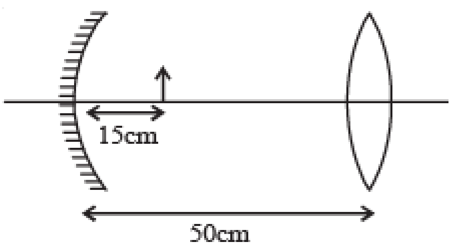
$\therefore \quad \sqrt{d^2+x^2} = 3m\lambda$

$\therefore \quad d^2+x^2 = 9m^2\lambda^2$

$\therefore \quad x^2 = 9m^2\lambda^2 - d^2$

$\therefore \quad p^2 = 9 \quad \Rightarrow \quad p = 3$

Q.6. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index 7/6, the magnification becomes M2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is (JEE Adv. 2015)



Ans. 7

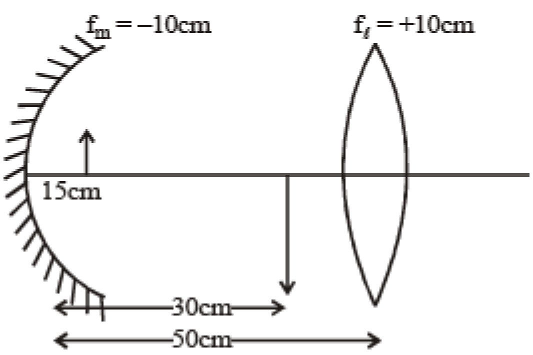
Solution. Applying mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-10}+\frac{1}{15}$$

$$\therefore \quad \frac{1}{v}=\frac{-15+10}{150}=\frac{-5}{150}=\frac{-1}{30}$$

$\therefore \quad v = -30\text{cm}$



For convex lens u = $|2f_\ell|$

Therefore image will have a magnification of 1.

When the set – up is kept in a medium

The focal length of the lens will change

$$\frac{\frac{1}{f_\ell}}{\frac{1}{f'_\ell}} = \frac{\left(\frac{n_\ell}{n_s}-1\right)}{\left(\frac{n_\ell}{n'_s}-1\right)} \Rightarrow \frac{f'_\ell}{10} = \frac{\left[\frac{1.5}{1}-1\right]}{\left[\frac{1.5}{7/6}-1\right]}$$

$\Rightarrow f'_\ell$ = 17.5 cm.

Applying lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f'_\ell}$

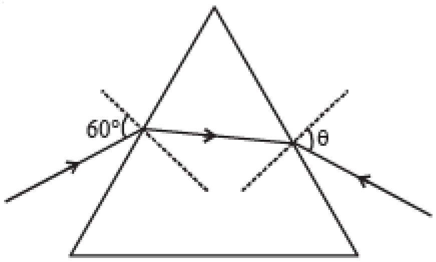
$$\therefore \quad \frac{1}{v}-\frac{1}{-20}=\frac{1}{17.5} \qquad \Rightarrow \quad v=140\text{ cm.}$$

M'_ℓ = Magnification by lens $=\frac{v}{u}=\frac{140}{-20}=-7$

$$\text{Now } \left|\frac{M_2}{M_1}\right| = \left|\frac{\text{Mmirror}\times M'_\ell}{\text{Mmirror}\times M_\ell}\right| = 7$$

Q.7. The monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle θ(n) with the normal (see the figure). For n = √3 the value of q is 60° and $\frac{d\theta}{dn}=m.$

The value of m is (JEE Adv. 2015)



Ans. 2

Solution. Here ∠MPQ + ∠MQP = 60°. If ∠MPQ = r then ∠MQP

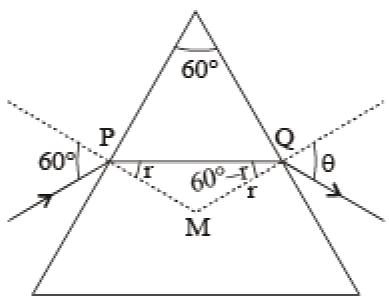
= 60 – r

Applying Snell's law at P

sin60° = n sin r ...(i)

Differentiating w.r.t 'n' we get

$$O = \sin r + n \cos r \times \frac{dr}{dn} \quad ...(ii)$$



Applying Snell's law at Q

sin θ = n sin (60° – r) ...(iii)

Differentiating the above equation w.r.t 'n' we get

$$\cos\theta \frac{d\theta}{dn} = \sin(60° - r) + n\cos(60° - r)\left[-\frac{dr}{dn}\right]$$

$$\therefore \cos\theta \frac{d\theta}{dn} = \sin(60° - r) - n\cos(60° - r)\left[-\frac{\tan r}{n}\right]$$

[from (ii)]

$\therefore \quad \frac{d\theta}{dn} = \frac{1}{\cos\theta}$ [sin (60° – r) + cos (60° – r) tan r] ...(iv)

From eq. (i), substituting n = √3 we get r = 30°

From eq (iii), substituting n = √3 , r = 30° we get θ = 60°

On substituting the values of r and θ in eq (iv) we get

$\frac{d\theta}{dn} = \frac{1}{\cos 60°}$ [sin 30° + cos 30° tan 30°] = 2